Genetic Algorithm for solving Dynamic Supply Chain Problem

Taher Taha Zaki [1], Naglaa Ragaa Saeid [2]

[1]Department of Operations Research, ISSR. Cairo University Cairo, Egypt

Abstract*— *The solution of dynamic supply chain problem is studied using both genetic algorithms and multistage dynamic programming. This is possible by employing Euler approximation method to approximate the differential of the variables. The problem is reformulated as unconstrained optimization problem which solved by genetic Algorithm and Multi Stages Dynamic Programming. The solution evaluation results from using dynamic programming and genetic algorithm is performed

***Keywords*— Dynamic Supply Chain, Genetic Algorithm, Multi Stages Dynamic Programming.**

I. INTRODUCTION

Now days the researchers are consternating in the problems of supply chain that varies with time. We introduced three research to solve this kind of problem using optimal control [4],[5],[6]. In this study we introduce genetic algorithm which is considered a tool of artificial intelligence [1]. [2] They studied a model of demand in dynamic setting of a multi product capacitated then by utilizing the KKT conditions to find the solution of their model .[12] introduced a model of supply chain and made his solution by utilizing GA. [11] proposed several applications of inventory model and made their solution using control theory in their book [9] applied a method of approximation to solve control problem by GA .

II. PRESUMPTION AND NOTATIONS

We suppose that corporation manufactures a one kind of a product. The company sales amount of product to holding company who sales it to end customer both the factory and distribution center have set stock preferred stock. Also the firm has set its production desired rate. The targets are to make the actual value of the stock and manufacture rate to get supply chain total cost at minimum as possible. T: Planning end time. $s_1(t)$: Rate of factory stock. $s_2(t)$: Rate of stock distribution center. $u(t)$: Rate of manufacture. $d(t)$: Rate of request. $s_1(0)$: Amount of initial Stock belongs to vendor $s_2(0)$: Amount of Initial factory Stock $\hat{u}$: Required rate of production. h_1 : holding cost of factory h_2: holding cost of distribution center. c: Expense of one unit of production. $\theta(t)$: spoiling rate. $\hat{s}_1$: Quantity of desired stock belongs to factory: $\hat{s}_2$ Quantity of desired stock belongs to distribution center.

Inventory deteriorating rate is obtained by : [9]

$$\theta(t) = ab\ t^{b-1}$$

where :

a : scale parameter $a > 0$

b : Shape parameter $b > 0$

III. MATHEMATICAL MODEL AND ANALYSIS

The objective function to be minimized is:

Minimize $j =$

$$\frac{1}{2}\int_0^T \left[\ h_1\left(s_1(t)-\hat{s}_1\right)^2 + h_2\left(s_2(t)-\hat{s}_2\right)^2 + c\left(u_v(t)-\hat{u}_v\right)^2\ \right] dt \quad (1)$$

Subject to

$$\dot{s}_1(t) = u(t) - d(t) - \theta(t)\, s_1(t) \qquad s_1(0) = s_1, \quad s_1 \succ 0 \quad (2)$$

$$\dot{s}_2(t) = -d(t) - \theta(t)\, s_2(t) \qquad s_2(0) = s_2, \quad s_2 \succ 0 \quad (3)$$

IV. DISCRETE APPROXIMATING OF LINEAR SYSTEM FIGURES AND TABLES

The genetic algorithm can be employed to run in linear time system. This can be done by dividing the continuous time system to remove the integration of objective function. Euler approximation method is used to discretize the system to approximate the differential of $s(t)$ [4],[8]

$$\dot{s}_1(t) = \frac{s_1(k+1) - s_1(k)}{m} \quad (4)$$

$$\dot{s}_2(t) = \frac{s_2(k+1) - s_2(k)}{m} \quad (5)$$

where m is discretization interval and $s(k) = s(km)$,the discrete of state variables of equations (2),(3) becomes

$$s_1(k+1) = s_1(k) + mu(k) - mD(k) - m\theta(k)\, s_1(k) \quad (6) \quad (6)$$

$$s_2(k+1) = s_2(k) + - mD(k) - m\theta(k)\, s_2(k) \quad (7) \quad (7)$$

The summation sign takes place with integration sign then equation (1) turns to:

$$j_k = \frac{1}{2}\sum_{k=k_0}^{k_f-1}[m h_1 (s_1(k)-\hat{s}_1)^2 + m_{h_2} (s_2(k)-\hat{s}_2)^2 + mc\,(u_v(k)-\hat{u}_v)^2] \quad (8)$$

V. IMPLEMENTATION OF GENETIC ALGORITHM FOR SOLVING DYNAMIC SUPPLY CHAIN MODEL

The solution of the problem in dynamic supply chain based on GA is analyzed using MATLAB. The affecting parameters used in the solution of the proposed method are described in table .1.

Table .1: GA Parameters

Type of population	double vector
Size of population	20
Generations	500
Rate of crossover	0.9
Rate of mutation	0.1
Selection Operator	Stochastic uniform

VI. IMPLEMENTATION OF DYNAMIC PROGRAMMING FOR SOLVING DYNAMIC SUPPLY CHAIN MODEL

The computations in dynamic programming are made by [5]

The equation to solve the dynamic programming is in the form (Forward recursive)

$$j_k(s(k)) = \min(f(s(k), u(k)) + j_{k-1}(s(k-1)) \quad \text{FOR} \quad k = 0,1,2,3,45 \quad (9)$$

VII. NUMERICAL EXAMPLE

In this section, we apply the proposed method to solve dynamic supply chain problem using GA. Table 2 contains the assumed parameters.

Table.2: Parameters given

The system parameters are assumed to have values in table below

Parameters	**Value**
a	1
b	1
$s_1(0)$	10
$s_2(0)$	15
$\hat{s}_1$	40
$\hat{s}_2$	25
C	\$1
h_1	\$1
h_2	\$1
$\hat{u}$	15
T	1

VIII. GENETIC ALGORITHM IMPLEMENTATION

The problem associated to this model is in the form :

Minimize j

$$= \frac{1}{2}\int_0^T [\ h_1 (s_1(t)-\hat{s}_1)^2 + h_2 (s_2(t)-\hat{s}_2)^2 + c\,(u_v(t)-\hat{u}_v)^2\ ]\ dt \quad (10)$$

Subject to

$$\dot{s}_1(t) = u(t) - d(t) - \theta(t)\, s_1(t) \qquad s_1(0) = s_1, \quad s_1 \succ 0 \quad (11)$$

$$\dot{s}_2(t) = -d(t) - \theta(t)\, s_2(t) \qquad s_2(0) = s_2, \quad s_2 \succ 0 \quad (12)$$

By applying the Euler approximation method, the problem can be converted to the following unconstrained problem which is solved by GA. All computations are executed by MATLAB 7.0.

$$\text{Minimize } j_k = \frac{1}{2}\sum_{k=k_0}^{k_f-1}[m h_1 (s_1(k)-\hat{s}_1)^2 + m h_2 (s_2(k)-\hat{s}_2)^2 + mc\,(u_v(k)-u_v)^2]$$

(13)

where

$$s_1(k+1) = s_1(k) + mu(k) - md(k) - m\theta(k)\, s_1(k) \quad (14)$$

$$s_2(k+1) = s_2(k) + -mD(k) - m\theta(k)\, s_2(k) \quad (15)$$

Setting $k = 0,1,2,3,4$ in the above equations (14), (15), we have

$$s_1(1) = s_1(0) + mu(0) - md(0) - m\theta(0)s_1(0) \quad (16)$$

$$s_1(2) = s_1(1) + mu(1) - md(1) - m\theta(1)s_1(1) \quad (17)$$

$$s_1(3) = s_1(2) + mu(2) - md(2) - m\theta(2)s_1(2) \quad (18)$$

$$s_1(4) = s_1(3) + mu(3) - md(3) - m\theta(3)s_1(3) \quad (19)$$

$$s_1(5) = s_1(4) + mu(4) - md(4) - m\theta(4)s_1(4) \quad (20)$$

$$s_2(1) = s_2(0) + -md(0) - m\theta(0)\, s_2(0) \quad (21)$$

$$s_2(2) = s_2(1) + -md(1) - m\theta(1)\, s_2(1) \quad (22)$$

$$s_2(3) = s_2(2) + -md(2) - m\theta(2)\, s_2(2) \quad (23)$$

$$s_2(4) = s_2(3) + -md(3) - m\theta(3)\, s_2(3) \quad (24)$$

$$s_2(5) = s_2(4) + -md(4) - m\theta(4)\, s_2(4) \quad (25)$$

Figure (1) shows the performance index j value and rate of production u_v against k for $k = 0,1,2,3,4$.

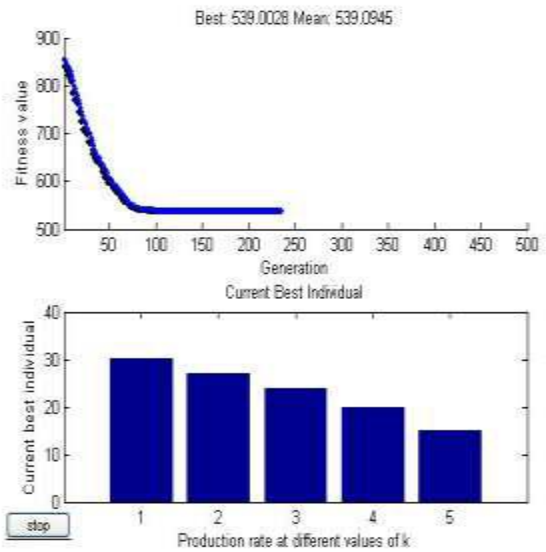

Fig.1: The values of objective function and production rate using GA at $k = 0,1,2,3,4$.

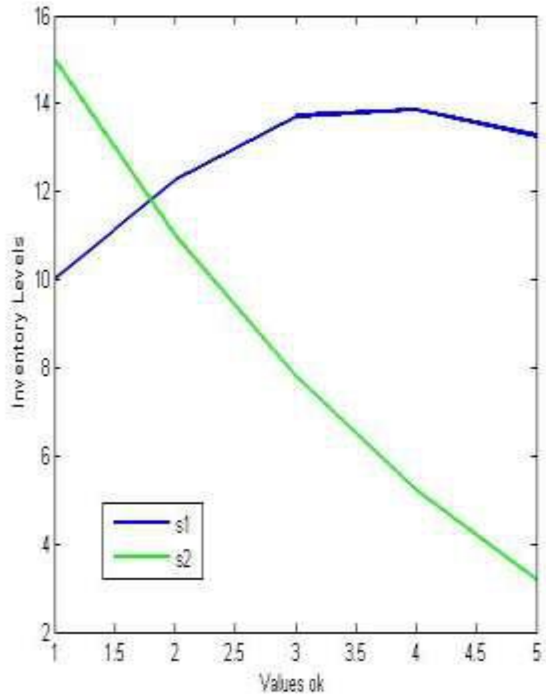

Fig.2: the values of state variables using GA at $k = 0,1,2,3,4$.

IX. IMPLEMENT USING DYNAMIC PROGRAMMING

In using the discretization of continuous-time processes, we employ the Euler approximation of a linear time invariant system (Naidu 2003). For m =0.2 to approximate the differential of $s_1(t)$, $s_2(t)$. Considering the constant demand rate $d(t)$ equal 5. The calculation of $s_1(t)$, $s_2(t)$ $\forall$ $k = 0,1,2,3,3,4$ is done by substitution in equations (16) to equation (25). $s_1(0)$=10 and $s_2(0)$ =15.The problem is divided to number of stages equal five. . We begin with first stage (1) ($k = 0,1$), then to the second stage $k = 2$ and so on and ending ate five stage $k = 5$.

Table.3: *State variables, control variables and objective function using dynamic programming for all values of* k.

k	s_1	s_2	$j_k + j_{k-1}$	u_v
0	10	15	109.9	25
1	12	11	218.1	25
2	13.48	7.81	327.28	25
3	14.78	5.24	431.4272	20
4	14.904	3.191	542.37	15

X. RESULTS PROVIDING USING GENETIC ALGORITHM AND DYNAMIC PROGRAMMING.

The running of this example using Dynamic Programming technique and genetic algorithm shows in Table 4.

Table .4: Results provided by, DP, GA.

k	DP	GA
	j	j
0	110.341	111.231
1	108.280	107.671
2	109.372	108.120
3	104.151	101.501
4	110.543	111.402

The GA which is recognized as one of the artificial intelligence techniques has been employed for solving the problem of SC. The outcome of employing GA to solve the problem of supply chain is ideal. In general GA is used to give almost global optimal solution.

XI. CONCLUSION

This study has intended to apply genetic algorithm and multi stages dynamic programming to find the solution problem of SC depending on the genetic algorithm which is known as one of tools of artificial intelligence has been used for solving optimal control problem.

ACKNOWLEDGEMENTS

An acknowledgement section may be presented after the conclusion, if desired.

REFERENCES

[1] Andrew Chipperfield , Peter Fleming , Hartmut Pohlheim , Hartmut Pohlhien , Carlos Fonseca. 2004, Genetic Algorithm Toolbox, MatWorks, Inc.

[2] ElodiaAdida. and Georgia Perakis . 2007, A Nonlinear Continuous Time Optimal Control Model of Dynamic Pricing and Inventory Control with No Backorders,Wiley Interscience. Vol. 54, No.2, PP. 767-795.

[3] Hamdi Ahmed Taha (1997), “Operations Research: An Introduction”, Sixth Edition, Perntice-Hall International Inc.

[4] Hegazy Zaher, Hegazy, Naglaa Rgaa S. and Taher Zaki. 2013, “an Optimal Control Theory Approaches to Solve Constrained Production and Inventory System”. International Journal of Advanced Research in Computer Science and Software Engineering, Vol.3, No. 12, PP. 665–670.

[5] Hegazy Zaher. and Taher ZAki. 2014, “Optimal control to solve production Inventory System in Supply Chain Management”, Journal of Mathematics Research, Vol. 6, No.4, PP. 109-117.

[6] Hegazy Zaher. and Taher Zaki 2015. “Application of Optimal Control Theory to Solve Three Stages Supply Chain Model”, Global Journal of Pure and Applied Mathematics, Vol.11, No. 2, PP. 167-17

[7] Morton I . Kamien and Nancy L. Schwartz, (1981), Dynamic Optimization, 2nd Edition, E lsevier North Holland Inc.

[8] Md. Azizul Baten and Anton Abdulubasah Kamil (2009), An Optimal Control Approach to Inventory Production System with Weibull Distributed Deterioration, Journal of Mathematics and Statistics, Vol. 5, No.3, PP.206-214.

[9] M. El-kady and N. El-sawy . (2013), Legendre Coefficients Method for Linear-Quadratic Optimal Control Problems by Using Genetic Algorithms ,International Journal of Applied Mathematical Research, 2 (1) (2013) 140-150

[10] Ogata Katsuhiko (2010), Modern Control Engineering, Fifth Edition, Printce Hall.

[11] Suresh.P Sethi,. and Thompson, Gerald .L. (2000), Optimal Control Theory, Applications to Management Science and Economics. 2nd Edn., Springer, USA., Inc.

[12] S. R. Sing and Tarun Kuna (2011), Inventory optimization in Efficient Supply Chain Management, International Journal of Computer Application in Engineering Sciences, Vol. 1 No. 4.,PP.107-118.