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How informed Receivers are influences the effect of Bayesian Persuasion: An example of Bank run

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Abstract— *This paper considers Bayesian persuasion game when receivers are partially informed andtheir behaviors influence each other. Receivers get signal independent of sender. And sender is fully informed about the state and signal receivers get. Sender sets a persuasion rule to give recommendation to receivers which plays role in communicating information of state and prompting cooperation between receivers.*
Keywords*— *Bayesian persuasion; Informed receiver; Bank run.

I. INTRODUCTION

Kamenica and Gentzkow (2011) introduce the concept of Bayesian persuasion. Sender can choose some information structure but cannot manipulate the result in the process of signal generating. They find there still exists opportunity for sender to benefit from persuade receiver on some conditions even receiver knows their information structure is designed by sender to maximize sender's utility. But in their paper, they do not consider how informed receiver are is going to influence the effect of persuasion. Receiver and sender share a common prior belief about the state, and sender renew their belief after observing the signal which comes from information structure designed by sender.

In our paper, we assume sender know the state and separate information design process into two parts. First, receiver has independent information structure about the state. The precision of the signal depends on how informed receiver is. Sender know the signal receiver get but cannot change it. Second, sender influences the receiver's posterior belief by setting a persuasion rule. Persuasion rule guarantees that sender gives receiver a recommendation about how to act in different state and with different signals. Once persuasion rule is set, sender must strictly comply with it. Hence, given prior belief, receiver's precision, and persuasion rule each signal induces some posterior distribution.

We analyze how informed receiver are influences the effect of persuasion. Generally, it is harder for sender to manipulate receiver's information structure or to persuade receiver to do what he wants if receiver is more informed. The logic behind this is that if receiver has more power to estimate the state it will be a smaller range of persuasion. More posterior beliefs cannot be achieved more precise receiver information is. Therefore, the manipulation power of sender on receiver's action is lower as precision increases. However, if there are many receivers and their actions are going to influence each other's payoff, the situation may be different. In one receiver game or receiver's payoff does not depend on other receivers' action, receiver just estimates the state precisely by his posterior belief which is induced by signal she gets. And receiver makes the best choice to maximize her payoff. Receiver does not care about others' belief and behavior, but only focus on the objective probability of the state. But in many-receiver game and their payoff depend on not only state but also other receivers' behavior. Thus, the persuasion rule does not just manipulate receivers' posterior belief and it recommends all receivers to behave cooperatively. Receivers not only derive posterior belief from her information structure and persuasion rule, also speculate other receivers' behavior through persuasion rule. The persuasion rule has two effect: first, it can reveal some information about the state; second, it can recommend receivers to behave cooperatively, avoid bad equilibrium and achieve good one. And sender can benefit from the higher precision of receivers. Higher precision makes receivers better know about the state which makes communication between sender and receivers more effective.

Our paper can be widely used in real world. For example, the authority knows some inside information but cannot directly communicate with people especially bad news. Because if they disclose the bad information, people may get panic which will cause greater loss. People get signal via TV, Internet or newspaper to speculate the reality. The precision of the signal is relevant to the report they learn from or their own ability. Report may be ambiguous about the event. And people may not be able to accurately understand real state by signals. They both influence the precision of receivers' information structure. People would like to update their belief by persuasion rule which give them opportunity to estimate state more precisely and predict other receivers' behaviors. Under persuasion rule, it is best to give recommendation that people have incentive to follow. If following persuasion rule is not best choice for people they may deviate from recommendation. Persuasion rule has only state estimation effect but no behavior prediction effect which means no cooperation equilibrium happens. It is obviously inefficient.If people are more informed, they can use the independent signal more effective. It also strengthens the communication effect between authority and people and benefit the authority. It is common to see that authority tries many ways to increase people's ability of understanding the country state, such that increasing average education level, controlling media.

In our paper, we present a specific bank run example to confirm our opinion. Bank's operating environment can turn to good or bad which influences the long-term return of bank's investment. In good state, depositors are willing to wait for long term. But in bad state, long-term return decreasing cannot compensate the risk that depositors bear. Worrying about happening of bad state, depositor may run in advance. Then, bank can set a persuasion rule signaling the real state of economy to depositors. It can decrease the probability of bank run.

In section 1, we give the introduction of the paper. In section 2, the literature review is presented. In section 3, we introduce general framework of the model. In section 4, we analyze the condition that receivers follow sender's recommendation and define an equilibrium of the model. We also interpret when sender can benefit from persuasion rule. In section 5, the bank run example shows sender can have receivers do as his recommendation and benefit from it. In section 6, we draw the conclusion.

II. LITERATURE REVIEW

Kamenica and Gentzkow (2011) introduce Bayesian persuasion. They consider a symmetric information model where sender chooses an information structure. Then information structure will generate a signal sent to receiver. Then receiver takes an action affecting utility of both sender and receiver. They derive conditions that signal strictly benefits the sender. They also derive the optimal signal from the concavification of sender' value function. And they characterize sender-optimal signals. Gentzkow and Kamenica (2014) consider signal cost and introduce a family of cost functions that is compatible with the concavification approach to deriving the optimal signal. Goldstein and Huang (2016) analyze a model that policymaker commits to abandon the regime whenever fundamentals are sufficiently week to decrease the attack and the regime survives more often. Alonso and Camara (2016) consider Bayesian persuasion when sender and receiver share different prior belief. Zhang and Zhou (2015) study how contest organizer design information structure to maximize players' devotion by Bayesian persuasion approach. Hedlund (2016) consider a privately informed sender with monotonic preferences. Bergemann and Morris (2016) analyze bank design information structure to minimize the probability of bank run. Their background is similar as ours. But they do not explain the bad state and good state. We separate investment choices into short-term and long-term which is like Diamond and Dybvig (1983). Depositors get payoff for short-term invest but bank can only achieve return in the long run. It cause that if too much depositor choose to run before bank get return long-term investors cannot get promised payoff. We assume long-term return is not certain but changes as the state. Based on Bergemann and Morris (2016)'s work, we further analyze the influence of how depositors are informed on persuasion rule.

III. MODEL

There is a two-state space $\Omega$with a typical state denoted ω. There are n receivers with action space $A=\{a_1,a_2\}$,andaction combination is denoted $\vec{a}$. Sender has a continuous utility function $v(\vec{a},\omega)$ that depends on Receiver's action and the state of the world. Receivers have a continuous utility function $u(\vec{a},\omega)$ that depends on Receivers' actions and the state of the world.Assume sender and receiver share a common prior belief $\mu_0 \in int(\Delta(\Omega))$of the world state $\omega_i, i\in\{1,2\}$. Sender is fully omniscient of the world state. Information structure of signal πconsists of a realization space S and a family of likelihood distribution $\pi=\{\pi(\cdot\,|\omega_i)\}_{i=1}^2$which depending on informed extent of receiver.Sender makes persuasion rule to recommend to receiver for choosing some actions by some probability. The rule depends on the world state ω_i and realized signal s_j , denoted as $R:(\omega_i,s_j)\to\rho_{\omega_i,s_j}(a),\omega_i\in\Omega,s_j\in S$, $\rho:\{a_1,a_2\}^n\to(0,1)$ .Kamenica and Gentzkow (2011) show that it is general to set cardinality of signal realization space same to state space, that is $|\Omega|=|S|=2$.

IV. OBEDIENT BAYESIAN PERSUASION GAME

And signal realization s and sender's recommendation $a^*$are observed by the sender and leads to a poster belief, denoted as μ_{s,a^*}.If there is no recommendation from sender, the poster belief of receiver after observing signal s is μ_s.We say $\mu_{s,a^*}$is Bayesian plausible if

$$\sum_a \mu_{s,a}\rho_{\omega_i,s}(a)=\mu_s.$$

Poster belief can be denoted as

$$\mu_{s,a}(\omega_i)=\frac{P_0(\omega_i)\pi(s|\omega_i)\rho_{\omega_i,s}(a)}{\sum_{\omega_j:\pi(s|\omega_j)>0,\rho_{\omega_j s}(a)>0}P_0(\omega_j)\pi(s|\omega_j)\rho_{\omega_j,s}(a)}$$

A persuasion rule is obedient if the receivers always have an incentive to follow the action recommendation from sender (Bergemann and Morris 2016). To make decision rule obedient, for any state ω_i, signal s_j, and any i,ithas to satisfy

$$E_{\mu_{s,a}}u_i(a,\omega_i)\geq\max_{a_i}E_{\mu_{s,a}}u_i((a_i,a_{-i},\omega).$$

Sender will choose a persuasion rule satisfying constraint above.And sender's utility maximization problem is

$$\max_R E_{\mu_0}E_sE_{\rho_{\omega,s}}v(a,\omega).$$

If the best decision rule $R^*\in e^i$ for $i\in\{1,\dots,N\}$, where e^irepresent unit vectors whose ith factor is 1 and others are zero, then we call the rule is degenerate. When decision rule is degenerated, it means sender recommends receivers to choose one best action with probability one with any signal and state. Let sender's expected utility with a rule decision $R=\rho_{\omega,s}$ be $\hat{v}(R)=\hat{v}(\rho_{\omega,s})\equiv E_{\mu_0}E_sE_{\rho_{\omega,s}}v(a,\omega)$.

If sender gives no recommendation about receiver's action, receivers' utility maximization problem is

$$\max_a E_{\mu_s}u(a,\omega_i),$$

where E_{μ_s} represents expectation utility of receivers according to receivers' poster belief of state afterobserving signal s

$$\mu_s(\omega_i)=\frac{P_0(\omega_i)\pi(s|\omega_i)}{\sum_{j:\pi(s|\omega_j)>0}P_0(\omega_j)\pi(s|\omega_j)}.$$

The best action for receiver is $a^*(\mu_s)\in\arg\max E_{\mu_s}u(a,\omega_i)$. And utility of sender is

$$E_{\mu_0}E_sv(a^*,\omega)$$

Let $\hat{v}(0)\equiv E_{\mu_0}E_sv(a^*,\omega)$.

Definition 1 If obedient condition is satisfied on persuasion rule R, every receiver will not deviate from sender's recommendation by knowing others will follow the recommendation. It constitutes a Bayesian Nash equilibrium.

Corollary 1 Sender benefits from making obedient persuasion rule$R(\omega, s)$ if and only if there exist Bayesian plausible distribution of posteriors μ_{s,a^*}such that

$$\hat{v}(\rho_{\omega,s}) > \hat{v}(0)$$

$$E_{\mu_{\omega_i s_j}} u(\vec{a}, \omega_i) \geq \max_a E_{\mu_{s,a^*}} u(\vec{a}, \omega)$$

Let V be concave closure of $\hat{v}$:

$$V(\rho_{\omega,s}) = \sup\{z|(\rho_{\omega,s}, z) \in co(\hat{v})\},$$

where $co(\hat{v})$denotes the convex hull of the graph of $\hat{v}$. If $(\rho'_{\omega,s}, z) \in co(\hat{v})$, then it can get $E\rho_{\omega,s} = \rho'_{\omega,s}$,rule decision $\rho'_{\omega,s}$can be achieved by mixing some rule decision and get value z.

Corollary2 The value of a rule decision $R = \rho_{\omega,s}$ is $V(\rho_{\omega,s})$. Sender benefits from this rule if and only if $V(\rho_{\omega,s}) > \hat{v}(0)$.

V. BANK RUN EXAMPLE

Is there exists some rule decision that receivers have incentive to follow sender's recommendations which benefits sender? We present an example of bank run to show that optimal obedient rule decision does exist in some situations. Further, we are going to analyze how uninformed and informed receivers influence the effectivity of persuasion.

Suppose that a bank borrows short-term deposit and lend long-term loan, in different states bank will get different rewards. There are continuum depositors on $[0,1]$ and two states: bad and good(denoted by B and G).Depositor and bank share a common prior belief on state $P_0(G) = P_G$ .There are three stages: 0,1,and 2. Rank declares persuasion rule in stage 0 what recommendation they will give to depositors when they know the stage in stage 2.Rewards depend on how much money still in the bank in stage 1 and get paid in stage 2. If the state is bad, the return rate will be r_B. If the state is good, the return rate will be r_G. The short-term return rate is r such that $1 < r_B < r < r_G$and $P_G r_G + (1 - P_G) r_B > r$. Bank wants to minimize the probability to run and depositors want to maximize their payoff.

The timing of the game is as follows. Bank sets a persuasion rule that gives a probability of stay recommendation for any combination of signal and state.

In stage 0, depositors(receivers) deposit their money into bank. Bank(sender) declares a persuasion rule which will be strictly obey.

In stage 1, depositors get a signal about the state and receive the recommendation coming from bank according to ex-ante persuasion rule. Then they decide to either run(r) or stay(s). And bank knows what signal depositors get and the world state.

In stage 2, the loan reward is given to depositors who choose to stay in stage 1 in average.

Uninformed depositors

If depositors are uninformed which means their get no signal in stage 1 but only receive bank's recommendation. Then the persuasion rule set by bank can be write as (ρ_G, ρ_B). To make persuasion rule obedient, some condition must be satisfied. When get stay recommendation, the depositor will then have an incentive to stay if

$$P_G \rho_G r_G + (1 - P_G) \rho_B r_B \geq P_G \rho_G r + (1 - P_G) \rho_B r, \tag{1}$$

and when get run recommendation, the depositor will then have an incentive to run if

$$P_G (1 - \rho_G) \times 1 + (1 - P_G)(1 - \rho_B) \times 1 \geq 0. \tag{2}$$

Since $P_G \in (0,1)$ and $\rho_G \in [0,1]$, (2) is always satisfied. (1)can be transformed to

$$\rho_G \geq \frac{1 - P_G}{P_G} \cdot \frac{r - r_B}{r_G - r} \cdot \rho_B.$$

In obedient persuasion rule, bank aim to increaseρ_G, ρ_G as they can. Therefore, it is optimal to let $\rho_G = 1$and $\rho_B = \frac{P_G}{1-P_G} \cdot \frac{r_G - r}{r - r_B}$.

With persuasion rule $(1, \frac{P_G}{1-P_G} \cdot \frac{r_G - r}{r - r_B})$, the probability to stay$P_s$is

$$P_s = P_G \rho_G + (1 - P_G) \rho_B = P_G + P_G \frac{r_G - r}{r - r_B}.$$

Therefore, bank run will not happen with probability P_s.

Informed depositors

Now, suppose that depositors receive information, independent of bank. We assume that there are two kind of signals: good(s_g)or bad(s_b). Depositors receive a correct signal with probability q, that is $\pi(s_g|G) = q, \pi(s_s|s) = q, q > \frac{1}{2}$.

The depositor's information will act like a constraint on the ability of the bank to influence the depositors' action, since bank has less control over the depositors' information. In this enriched setting, a persuasion rule can be represented by probability that bank recommends depositors to say, as a function of both the state and the signal. We denote $\rho_{\delta t}$ as the probability of staying in state $\delta \in \{G, B\}$ and signal is $t \in T = \{g, b\}$. The persuasion rule is now described by the quadruple $(\rho_{Gg}, \rho_{Gb}, \rho_{Bg}, \rho_{Bb})$.

The analysis of the informed depositor case will depend on what bank knows about depositors' information. We assume that bank knows what initial information depositors receive and gives recommendation condition on signals. It is a suitable assumption because bank may not be able to change the media's report about their financial statement but know it as well as depositors. And bank can release different signal to depositors to change their action.

Now we turn to obedient rule constraint. If depositors observe a good signal and stay recommendation, the constraint is

$$\frac{P_G q \rho_{Gg}}{P_G q \rho_{Gg} + (1 - P_G)(1 - q)\rho_{Bg}} r_G + \frac{(1 - P_G)(1 - q)\rho_{Bg}}{P_G q \rho_{Gg} + (1 - P_G)(1 - q)\rho_{Bg}} r_B \geq r. \qquad (3)$$

If depositors observe a good signal and run recommendation, the constraint is

$$P_G q(1 - \rho_{Gg}) r_G + (1 - P_G)(1 - q)(1 - \rho_{Bg}) r_B \geq 0. \qquad (4)$$

If depositors observe a bad signal and stay recommendation, the constraint is

$$\frac{P_G(1 - q)\rho_{Gb}}{P_G(1 - q)\rho_{Gb} + (1 - P_G) q \rho_{Bb}} r_G + \frac{(1 - P_G) q \rho_{Bb}}{P_G(1 - q)\rho_{Gb} + (1 - P_G) q \rho_{Bb}} r_B \geq r. \qquad (5)$$

If depositors observe a bad signal and run recommendation, the constraint is

$$P_G(1 - q)(1 - \rho_{Gb}) \times 1 + (1 - P_G) q (1 - \rho_{Bb}) \times 1 \geq 0. \qquad (6)$$

(4), (6)is satisfied naturally. From (3)and (5), we have

$$P_G q \rho_{Gg}(r_G - r) \geq (1 - P_G)(1 - q)\rho_{Bg}(r - r_B), \qquad (7)$$

$$P_G(1 - q)\rho_{Gb}(r_G - r) \geq (1 - P_G) q \rho_{Bb}(r - r_B). \qquad (8)$$

In order to maximize the probability to stay $P_s' = P_G \pi_g \rho_{Gg} + P_G \pi_b \rho_{Gb} + (1 - P_G)\pi_g \rho_{Bg} + (1 - P_G)\pi_b \rho_{Bb}$, where $\pi_g = P_G q + (1 - P_G)(1 - q)$, $\pi_b = P_G(1 - q) + (1 - P_G) q$, we set

$$\rho_{Gg} = 1, \rho_{Gb} = 1.$$

Substitute into (7), (8), we have

$$\rho_{Bg} = \frac{P_G q(r_G - r)}{(1 - P_G)(1 - q)(r - r_B)},$$

$$= \rho_{Bb} \frac{P_G(1 - q)(r_G - r)}{(1 - P_G) q (r - r_B)}.$$

Then, substitute $\rho_{Gg}, \rho_{Gb}, \rho_{Bg}, \rho_{Bb}$into P_s', we have

$$P_s' = P_G + (1 - P_G)\left\{[P_G q + (1 - P_G)(1 - q)]\frac{P_G q(r_G - r)}{(1 - P_G)(1 - q)(r - r_B)} + [P_G(1 - q) + (1 - P_G) q]\frac{P_G(1 - q)(r_G - r)}{(1 - P_G) q (r - r_B)}\right\}.$$

Proposition 1 When depositors are informed, the probability to run is lower than unformed if the bank declares the most preferred persuasion rule.

Proof. $P_s' = P_G + [P_G q + (1 - P_G)(1 - q)]\frac{P_G q(r_G - r)}{(1 - q)(r - r_B)} + [P_G(1 - q) + (1 - P_G) q]\frac{P_G(1 - q)(r_G - r)}{q(r - r_B)}$

$$= P_G + P_G\left\{[P_G q + (1 - P_G)(1 - q)]\frac{q}{1 - q} + [P_G(1 - q) + (1 - P_G) q]\frac{1 - q}{q}\right\}\frac{r_G - r}{r - r_B}$$

$$= P_G + P_G\left\{1 - P_G + P_G\left[\frac{q^2}{1-q} + \frac{(1-q)^2}{q}\right]\right\}\frac{r_G - r}{r - r_B}$$

$$> P_G + P_G\frac{r_G - r}{r - r_B} = P_s.$$

Let $q = \frac{1}{2}$, we have $P_s' = P_s$. Depositors are totally uninformed. Hedlund(2016) defines the information structure $\pi'$is more precise than πif for any signal $t \in T$, either $\pi'(t|G) \leq \pi(t|G) \leq \pi(t|B) \leq \pi'(t|B)$ or $\pi'(t|B) \leq \pi(t|B) \leq \pi(t|G) \leq \pi'(t|G)$. If $\pi \neq \pi'$, then $\pi'$is strictly more precise than π. We say depositors are more informed if their information structure is more precise. And depositors' precision of signal increases as $q$increases.

Proposition 2 Given the best persuasion rule declared by bank, the probability to run is lower if the depositors are more informed.

The proof of proposition is obvious because P_s' is monotonic increasingly to q, for $q > \frac{1}{2}$.

The precision change has no influence on $\rho_{Gg}$and ρ_{Gb}. Bank will always recommend stay when state is good irrelevant to receivers' precision. But the precision increasing tends to increase ρ_{Bg} and decrease ρ_{Bb}. Depositors with high precision are more willing to trust their signal. Thus, when they get good signal they are easily to be persuaded to stay. However, when they get bad signal they are more likely to run. The former is stronger than the later. Therefore, the probability to run decreases as precision grows.

VI. CONCLUSION

In this paper, we build Bayesian persuasion model in a different way. Not like others, we model the receiver' power to predict. We assume receiver can observe a signal independent of sender. Sender influence receiver's information structure by giving a recommendation. Sender is fully informed and his recommendation is rely on the state and signal. He declares a persuasion rule which he must obey. And receiver updates her belief by the signal she observes and sender's recommendations. What influence sender can make on receiver's posterior distribution is relevant to receiver's informed extent. In many models, sender benefits from persuasion less if receiver is more informed. Because the effect of Bayesian persuasion is changing receiver's information structure and manipulating receiver's behavior. Once receiver is more informed, there left less space for sender to persuade. But if there are many receivers and their payoff is decided by strategy combination, situation may be different. Because the persuasion process has not only manipulation effect but also prediction effect. Sender set persuasion rule to help receivers predict each other's behavior, make sure no one deviates from recommendation, andachieve cooperation outcome.

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