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Approximation schemes for Impulse Control Systems with Unbounded Cost Functional

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***Abstract*— *The impulse control problem with unbounded cost functional is different from the bounded case. We prove comparison result of viscosity solution by converting the unbounded value function into bounded one by suitable transformation and prove that the value function is the unique viscosity solution of the related first order Hamilton–Jacobi quasi-variational inequality (QVI). We construct a time discretization scheme for the QVI and prove that the approximate value function exists, that it is the unique solution of the approximate QVI. We also prove that the solution of the time discretization scheme converges to the viscosity solution of the QVI, when the discretization step goes to zero. The optimal control of the time discrete system determined by the corresponding dynamic programming is a minimizing sequence of the optimal feedback control of the impulse control problem.*
Keywords*— *Impulse control system; Viscosity solution; Hamilton-Jacobi quasi-variational inequality; Uniqueness; Approximation schemes

I. INTRODUCTION

The optimal feedback control can be constructed based on the approximation of viscosity solution, if the convergence of the approximation is proved. We refer to [1, 3, 4, 10-12, 24] for studies of the existence and uniqueness of the viscosity solution to HJB equation. More recent efforts in solving numerically the HJB equation for finite-dimensional control problems can be found in [8, 9, 13, 15, 21- 23, 27, 28]. In recent work [17] author has studied regularity results for the value function of the infinite horizon impulse control problem with bounded cost functions, and shown that it is the unique bounded and uniformly continuous viscosity solution of the associated QVI. Also has shown that the approximate value function converges locally uniformly, towards the value function of the impulse control problem, when the discretization step h goes to zero. In the case of finite horizon impulse control [6], the discretization is based on Euler scheme with step h. In the previous literature, authors in [5] use the approach of changing the original infinite horizon impulse control problem into another one equivalent without impulses, by adding one state variable, in order to use the classic dynamic programming theory.

In [19], where weaker assumptions on data are examined, the authors consider some general impulse control problems with a stopping time control. They assume that the number of impulses is finite and that both continuous and impulse controls have values in compact sets. The authors prove that the approximate value function converges uniformly to the value function of the impulse control problem. In [20], the authors study the same problem in the stationary case.

Authors studied the impulse optimal control problem with unbounded cost and shown that the value function of the stopping time problem is the unique viscosity solution in a suitable subclass of continuous functions, of the associated QVI. [25]

In an earlier work, Dharmatti and Ramaswamy [14] have studied model of hybrid control system in which both discrete and continuous controls are involved and shown that value function is unique bounded and Hölder continuous viscosity solution of associated quasi-variational inequality (QVI). The case of hybrid control problem with unbounded cost functional was studied in Barles et al. [2].

In [18, 26], the authors have shown a numerical scheme for constructing hybrid control in feedback form for the control problem of the type studied in infinite horizon case.

In this paper, we study a deterministic infinite horizon, mixed continuous and impulse control problem with unbounded cost functions.

We prove that the value function of the control problem is the unique viscosity solution of the related first order Hamilton–Jacobi quasi-variational inequality.

The major difficulties arise due to unboundedness of value function.

The proof in the case of bounded value function will not work for unbounded cost functional.

In the previous literature, authors in [1] use the approach of transforming the original HJB equation for the minimum time problem into another equation with the good structure to apply the comparison results, by the exponential transformation, also named the Kruzkov transformation.

To treat the case of unbounded value functions, we convert these value functions to bounded ones using exponential transformation and prove comparison result.

Then in a suitable function class, we prove that value functions are unique viscosity solutions of QVI's. We also construct a time discretization scheme for the QVI and prove that the approximate value function exists, that it is the unique solution of the approximate QVI.

We also prove that the solution of the time discrete scheme converges to the viscosity solution of the QVI, when the discretization step goes to zero, and prove that the optimal control of the time discrete system determined by the corresponding dynamic programming is a minimizing sequence of the optimal feedback control of the impulse control problem.

II. SETTING OF THE PROBLEM AND ASSUMPTIONS

We consider a mixed continuous and impulse control problem:

$$\begin{cases} y(\tau) = f(y(\tau), u(\tau)), \tau \in [\tau_{k-1}, \tau_k) \\ y(0) = x \\ y(\tau_k^+) = g(y(\tau_k^-), w_k) \end{cases} \quad (1)$$

, where $y(\cdot)$ is a state, $u(\tau)$ being the continuous control, $(\{\tau_k\},\{w_k\})$ being the impulse control, $(\tau_k)_{k\in N}$ is a non-decreasing sequence of positive reals which satisfies: $\tau_k \to +\infty$ when $k \to +\infty$ and $\tau_0 = 0$.

Also, $u(\cdot) \in U = \{u(\cdot) : [0,\infty) \to U \mid u : measurable\}$, $w_k \in W$; and $U \subset R^m, W \subset R^p$ are both compact sets. The control system can be transferred into the state of predefined set $D \subset R^n$ at any time by the state transition mapping $g : R^n \times W \to D$ according to the controller's decision and $y(\tau_k^-), y(\tau_k^+)$ denotes respectively state before and after jump of the system at time τ_k.

We denote by $\theta = (u(\cdot), \{\tau_k\}, \{w_k\})$ the hybrid control and by $y_x(\tau)$ the trajectory starting from the initial state x at time *t*. If we denote all possible hybrid control set by Θ, we associate the cost defined by

$$J(x,\theta) = \int_0^\infty l(y_x(\tau), u(\tau)) e^{-\lambda\tau} d\tau + \sum_{l\in N} \rho(y_x(\tau_i^-), w_i) e^{-\lambda\tau_i} \quad (2)$$

, where $\lambda > 0$ is the discount factor.

The value function v of the impulse control problem is defined as

$$v(x) = \inf_{\theta\in\Theta} J(x,\theta).$$

We make the following assumptions for the impulse control problem:

(A1): $f : R^n \times U \to R^n, g : R^n \times W \to D$ is respectively continuous and satisfies the following conditions:

$$\exists L_f > 0, \forall x, y \in R^n, \forall u \in U, \|f(x,u) - f(y,u)\| \le L_f \|x - y\|$$

$$\exists L_g > 0, \forall x, y \in R^n, \forall w \in W, \|g(x,w) - g(y,w)\| \le L_g \|x - y\|$$

$$\exists M_f > 0, \forall x \in R^n, \forall u \in U, \|f(x,u)\| \le M_f$$

(A2): $l : R^n \times U \to R$, $\rho : R^n \times W \to R$ is respectively nonnegative continuous function and satisfies the following conditions:

$$\exists L_l > 0, \forall x, y \in R^n, \forall u \in U, |l(x,u) - l(y,u)| \le L_l \|x - y\|$$

$$\exists C > 0, \forall x \in R^n, \forall u \in U, |l(x,u)| \le C(1 + \|x\|)$$

$$\exists L_\rho > 0, \forall x, y \in R^n, \forall w \in W, |\rho(x,w) - \rho(y,w)| \le L_\rho \|x - y\|$$

$$\exists m_\rho > 0, \forall x \in R^n, \forall w \in W, m_\rho(1 + \|x\|) \le \rho(x,w) \le C(1 + \|x\|)$$

(A3): $D \subset R^n$ is a nonempty bounded closed set and satisfies

$$\exists \overline{R} > 0, \forall x \in D, \|x\| < \overline{R}.$$

Remark 2.1. In [25], the authors consider impulse control systems, mixed continuous and impulse control:

$$\begin{cases} y(\tau) = f(y(\tau), u(\tau)), \tau \in [\tau_{k-1}, \tau_k) \\ y(0) = x \\ y(\tau_k^+) = y(\tau_k^-) + w_k \end{cases}$$

They assume that the discount factor $\lambda > L_f$, where L_f is the same in (A1).

III. LOCALLY LIPSCHITZ CONTINUOUS OF THE VALUE FUNCTION AND THE UNIQUENESS OF THE VISCOSITY SOLUTION

For the value function, the following fundamental equation of dynamic programming hold:

Lemma 3.1.

1) For all $s > 0$, we have

$$v(x) = \inf_{\vartheta \in \Theta} \left\{ \int_0^s l(y_x(\tau), u(\tau)) e^{-\lambda \tau} d\tau + \sum_{\tau_k < s} \rho(y_x(\tau_k^-), w_k) e^{-\lambda \tau_k} + e^{-\lambda s} v(y_x(s)) \right\} \tag{3}$$

2) There exists $s_0 > 0$ such for all $0 < s < s_0$, we have

$$v(x) = \min \left\{ Mv(x), \inf_{u(\cdot) \in U} \left(\int_0^s l(y_x(\tau), u(\tau)) e^{-\lambda \tau} d\tau + e^{-\lambda s} v(y_x(s)) \right) \right\} \tag{4}$$

, where $Mv(x) = \inf_{w \in W} \{ v(g(x,w)) + \rho(x,w) \}$.

Theorem 3.2. Assume (A1) and (A2) for the impulse optimal control problem Eq. (1) and Eq. (2). Then, for the value function $v(t,x)$, we have

$$\exists \overline{C} > 0, \forall x \in R^n, v(x) \le \overline{C}(1 + \|x\|).$$

Moreover, if $\lambda > L_f, L_g \le 1$, the value function v is Lipschitz continuous. That is, the following holds:

$$\exists C_1 > 0, \forall x, z \in R^n, |v(x) - v(z)| \le C_1 \|x - z\| . \tag{5}$$

Remark 3.3. In [17], the authors proved that the value function is Lipschitz continuous when $\lambda > L_f, L_g \le 1$ for the infinite time impulse problem with unbounded cost functional.

Now, for arbitrary $\alpha > 0$, denote spaces of $u \in C(R^n)$ such that

$$\lim_{\|x\| \leftarrow \infty} u(x) e^{-\alpha\|x\|} = 0$$

as $E_\alpha(R^n)$. By Eq. (2), the value function v satisfies $v \in E_\alpha(R^n)$.

Definition 3.4. Let be $u: R^n \to R$ continuous. For arbitrary $\psi \in C^1(R^n)$, at local minimum point (local maximum point) $x \in R^n$ of $u - \psi$ if

$$\max\{u(x) - Mu(x), \lambda u(x) + H(x, D\psi(x))\} \geq 0$$

$$(\max\{u(x) - Mu(x), \lambda u(x) + H(x, D\psi(x))\} \leq 0)$$

, we say that u is a viscosity super-solution (sub-solution) of Hamilton-Jacobi QVI:

$$\max\{u(x) - Mu(x), \lambda u(x) + H(x, D\psi(x))\} = 0, x \in R^n. \qquad (6)$$

If u is both a super and sub-solution of Eq. (6), we say that u is a viscosity solution of Eq. (6), where

$$Mu(x) = \min_{w \in W}\{u(g(x,w)) + \rho(x,w)\}.$$

By lemma 3.1, we obtain that the value function $v(x)$ is a viscosity solution of Eq. (6) for the impulse optimal control problem Eq. (1) and Eq. (2), where

$$H(t,x,p) = \sup_{u \in U}\{-f(t,x,u) \cdot p - l(t,x,u)\}.$$

Now, let's consider a function $u(x) = v(x)e^{-\alpha\xi(x)}$ when $v(x)$ is a value function of impulse optimal control problem Eq.(1) and Eq.(2)

, where $\xi : R^n \to R^+$ is a smooth function such that $\|D\xi(x)\| \leq 1, x \in R^n$

$$\text{and } \xi(x) = 0, \|x\| \leq \overline{R}, \xi(x) = (1 + \|x\|^2)^{1/2}, \|x\| > 2\overline{R}.$$

$\overline{R}$ is the same in the assumption (A3).

Lemma 3.5. Assume that $v(x)$ is a viscosity solution of Eq. (7). Then for any $\alpha > 0$, $u(x) = v(x)e^{-\alpha\xi(x)}$ is a viscosity solution of Hamilton-Jacobi QVI

$$\max\{u(x) - \overline{M}u(x), \lambda u(x) + \overline{H}(x, Du(x))\} = 0 \qquad (7)$$

, where $\overline{M}u(x) = \min_{w \in W} e^{-\alpha\xi(x)}\{u(g(x,w)) + \rho(x,w)\}$,

$$\overline{H}(x, Du(x)) = \sup_{u \in U}\{-f(x,u) \cdot (Du(x) + \alpha u(x) D\xi(x)) - e^{-\alpha\xi(x)} l(x,u)\}.$$

Remark 3.6. Let $v(x)$ be a value function of the impulse optimal control problem Eq. (1) and Eq. (2). Then, instead Eq. (2) $u(x) = v(x)e^{-\alpha\xi(x)}$ is a value function when the cost function is

$$\overline{J}(t,x,\theta) = \int_0^\infty l(y_x(\tau), u(\tau)) e^{-\lambda\tau} e^{-\varepsilon\xi(x)} d\tau + \sum_{k \in N} \rho(y_x(\tau_k^-), w_k) e^{-\lambda\tau_k} e^{-\alpha\xi(x)}.$$

Lemma 3.7. Let $u(x)$ be Lipschitz continuous. Then function $\overline{M}u(x)$ is also bounded Lipschitz continuous.

Theorem 3.8. Assume (A1) - (A3) hold for the impulse optimal control problem Eq. (1) and Eq. (2). Let $v_1, v_2 \in E_\alpha(R^n)$ be respectively continuous viscosity sub and super solution of Eq. (7) for $0 < \alpha < \frac{\lambda}{M_f}$. Then we get $v_1(x) \le v_2(x), x \in R^n$.

Theorem 3.9. Let $v(x)$ be a value function of the impulse optimal control problem Eq. (1) and Eq. (2) and assume (A1)-(A3). Then, the value function $v(x)$ is the unique uniformly continuous viscosity solution of the Hamilton-Jacobi QVI Eq. (6).

IV. THE CONVERGENCE OF THE APPROXIMATION SCHEME FOR VALUE FUNCTION

Consider the time discretization case. Choose the time step $h(0 < h < h_0)$ and $t_j = jh, j = 0,1,2,...$, where $h_0 \le \min\{1, 1/\lambda\}$.

Then if we choose time step h small enough, we can consider that the operator's state jump time $\{\tau_k\}$ is approximately $\tau_k \approx t_{j_k}$. Then we have the discrete control system:

$$\begin{cases} y_{j+1} = y_j + hf(y_j, u_j), j = 0,1,2... \\ y_0 = x \\ y_{j_k} = g(y_{j_k}{}^-, w_k) \end{cases} \tag{8}$$

, where $y_j = y(t_j), u_j = u(t_j), t_j = jh, t_{j_k} = \tau_k = j_k h$ and $y_{j_k}{}^- = y_{j_k - 1} + hf(y_{j_k - 1}, u_{j_k - 1})$.

The cost functional and value function to the discrete system Eq. (8) are defined, respectively, by

$$J_h(x, \{u_j\}, \{\tau_k\}, \{w_k\}) = \sum_{j=0}^{\infty} hl(y_j, u_j) e^{-\lambda jh} + \sum_{j_k} \rho(y_{j_k}{}^-, w_k) e^{-\lambda j_k h} \to \min_{\{u_j\},\{\tau_k\},\{w_k\}} \tag{9}$$

, where $v_h(x) = \inf_{\{u_j\},\{\tau_k\},\{w_k\}} J_h(x, \{u_j\}, \{\tau_k\}, \{w_k\})$.

Lemma 4.1. The value function v_h satisfies

$$v_h(x) = \min[\inf_{u \in U}\{hl(x,u) + e^{-\lambda h} v_h(x + hf(x,u))\}, M^h v_h(x)], x \in R^h \tag{10}$$

, where $M^h v_h(x) = \min_w \{\rho(x,w) + v_h(g(x,w))\}$.

Theorem 4.2. Assume (A1) and (A2). Then v_h is non-negative and satisfies:

$$\exists \overline{C} > 0, \forall x \in R^n, v_h(x) \le \overline{C}(1 + \|x\|).$$

Moreover, for $\lambda > L_f, L_g \le 1$, v_h is Lipchitz continuous, i.e.

$$\exists \overline{C}_1 > 0, \forall x, z \in R^n, | v_h(x) - v_h(z) | \le \overline{C}_1 \|x - z\|. \tag{11}$$

Lemma 4.4. u_h, v_h are bounded and uniformly continuous in any compact subset $K \subset R^n (D \subset K)$.

Then the following statements hold.

1. $M^h v_h$ is bounded and uniformly continuous in K.
2. If $u_h \le v_h$, then $M^h u_h \le M^h v_h$.
3. When $0 \le \mu \le 1$, we have

$$\mu M^h u_h + (1-\mu) M^h v_h \le M^h (\mu u_h + (1-\mu) v_h).$$

Define the following operator:

$$(Tu_h)(x) := \min[\inf_{u \in U}\{hl(x,u) e^{-\alpha \xi(x)} + e^{-\lambda h} e^{\alpha(\xi(x + hf(x,u)) - \xi(x))} u_h(x + hf(x,u))\}, \overline{M}^h u_h(x)].$$

Then, (10) can be rewritten as

$$u_h(x) = (Tu_h)(x), x \in R^n.$$

By the definition of the operator and lemma 4.4, we get the followings.

Lemma 4.5. u_h, v_h are bounded and uniformly continuous in any compact subset $K \subset R^n (D \subset K)$.

1. Tv_h is bounced and uniformly continuous in K.
2. If $u_h \le v_h$, then $Tu_h \le Tv_h$.
3. When $0 \le \mu \le 1$, we have

$$\mu Tu_h + (1-\mu)Tv_h \le T(\mu u_h + (1-\mu)v_h).$$

Theorem 4.6. Assume (A1) - (A3) and $0 < h < h_0, \lambda > L_f$.

Then the time discretization Hamilton-Jacobi QVI Eq. (10) has the unique solution in R^n.

Theorem 4.7. Let $v(x)$ be the value function of the impulse optimal control problem Eq. (1), Eq. (2). If $0 < h < h_0, \lambda > L_f$, $L_g \le 1$, then v_h converges to v uniformly as $h \to 0$ for any compact subset $K \subset R^n (D \subset K)$.

V. APPROXIMATION OF THE OPTIMAL FEEDBACK CONTROL

Define the following equation for the value function v_h of the time discretization problem Eq. (8) and Eq. (9).

$$m_j = \inf_{u \in U}\{hl(y_j,u) + e^{-\lambda h} v_h(y_j + hf(y_j,u))\} = hl(y_j,u_h^*(y_j)) + e^{-\lambda h} v_h(y_j + hf(y_j,u_h^*(y_j))) \quad (12)$$

$$\omega_j = \min_w \{\rho(y_j,w) + v_h(g(y_j,w))\} = \rho(y_j,w^*(y_j)) + v_h(g(y_j,w^*(y_j))) \quad (13)$$

Construct an admissible control $(\{u_j\},\{\tau_k\},\{w_k\})$ for the time discretization problem as follows.

If $m_j \le \omega_j, j = \overline{0, j_1 - 1}$, we set $u_{hj}^* = u_h^*(y_j)$ and we get $\{y_j\}, j = \overline{0, j_1}$ from

$$y_{j+1} = y_j + hf(y_j, u_{hj}^*), y_0 = x.$$

If $m_j > \omega_j, j = \overline{0, j_1 - 1}$, we set $j_1 = 0$.

And if $m_{j_1} > \omega_{j_1}$, we set $\tau_1 = j_1, w_1^* = w^*(y_{j_1})$ and state before jump $y_{j_1}^-$ moves to $y_{j_1}^+ = g(y_{j_1}^-, w_1^*)$ by the state transition function and we get $u_h^*(y_{j_1}{}^+)$ from Eq. (12) and Eq. (13). Then, represent $y_{j_1}^+$ and $u_h^*(y_{j_1}^+)$ as $y_{j_1}, u_{hj_1}^* = u_h^*(y_{j_1})$.

Consider the control sets $\{u_{kj}^*\},\{\tau_k\},\{w_k^*\}$ obtained from the Eq. (12) and Eq. (13) and

$$\begin{cases} y_{j+1} = y_j + hf(y_j, u_{hj}^*), j = 0,1,2\ldots \\ y_0 = x, x \in R^n \\ y_{j_k} = g(y_{j_1}^-, w_k^*) \end{cases} \quad (14)$$

Lemma 5.1. Let v_h be a value function for the time discretization Eq. (8) and Eq. (9).

Then we get

$$v_h(x) = J_k(x, \{u_{kj}^*\}, \{\tau_k\}, \{w_k^*\}).$$

Theorem 5.2. Suppose that $0 < h < h_0, \lambda > L_f, L_g \le 1$ and v_h is the value function of the time discretization problem Eq. (8)

and Eq. (9). If $x \in K$ for every compact subset $K \subset R^n$, suppose that

$$u_h^*(\tau) = u_{hj}^*, \tau \in [t_j, t_{j+1}), j = 0,1,2,...$$

for control sets $\{u_{hj}^*\}, \{\tau_k\}, \{w_k^*\}$ obtained from Eq. (12), Eq. (13) and Eq. (14).

Then we have

$$\lim_{h \to 0} J(x, \{u_k^*\}, \{\tau_k\}, \{w_k^*\}) = v(x).$$

VI. CONCLUSIONS

We prove the comparison result of viscosity solution by converting the unbounded value function into bounded one by suitable transformation and prove that the value function is the unique viscosity solution of the related first order Hamilton–Jacobi quasi-variational inequality.The convergence of the approximation schemes for value function and optimal feedback control based on discrete time Dynamic Programming are proved.

REFERENCES

[1] Bardi M., Capuzzo Dolcetta I., 2008. Optimal Control and Viscosity Solutions of Hamilton-Jacobi- Bellman Equations, Birkhäuser, Boston, 586.

[2] Barles G., Dharmatti S., Ramaswamy. M., 2010. Unbounded viscosity solutions of hybrid control systems, ESAIM Control Optim. Calc., 16 (1), 176–193.

[3] Barles G., Souganidis P.E., 1991. Convergence of approximation schemes for fully nonlinear second order equations. J. Asymptot. Anal. 4, 271–283.

[4] Barron E. N., 1990. Application of viscosity solutions of infinite-dimensional Hamilton–Jacobi–Bellman equations to some problems in distributed optimal control. J. Optim. Theory Appl. 64, 245–268.

[5] Camilli F., Falcone M., 1997. Analysis and approximation of the infinite-horizon problem with impulsive controls, Autom. Remote Control 58, 1203–1215.

[6] Camilli F., Falcone M., 1999. Approximation of control problems involving ordinary and impulsive controls, ESAIM: Control Optim. Calc. Var. 4, 159–176.

[7] Camill F., Falcone M., 1995. An approximation scheme for the optimal control of diffusion processes. RAIRO Modélisation Mathématique et Analyse Numérique 29, 97–122,

[8] Capuzzo Dolcetta I., On a discrete approximation of the Hamilton–Jacobi equation of dynamic programming, Appl. Math. Optim. 10, 367–377, 1983.

[9] Capuzzo Dolcetta I., Ishii H., 1984. Approximate solutions of the Bellman equation of deterministic control theory, Appl. Math. Optim. 11, 161–181.

[10] Crandall M.G., Evans L.C., Lions P.L., 1984. Some properties of viscosity solutions of Hamilton–Jacobi equations. Tran. Amer. Math. Soc. 282, 487–502.

[11] Crandall M.G., Evans L.C., Lions P.L., 1992. User's guide to viscosity solutions of second order partial differential equations. Bull. Amer. Math. Soc. 27, 1–67.

[12] Crandall M.G., Lions P.L., 1983. Viscosity solutions of Hamilton–Jacobi equations. Tran. Amer. Math. Soc. 277, 1–42.

[13] Crandal M.G., Lions P.L., 1984. Two approximations of solutions of Hamilton–Jacobi equations. Math. Comp. 43, 1–19.

[14] Dharmatti S., Ramaswamy M., 2005. Hybrid control systems and viscosity solutions, SIAM J. Control Optim., 44, 1259-1288.

[15] Falcone M., 1987. A numerical approach to the infinite horizon problem of deterministic control theory. Appl.Math. Optim. 15, 1–13.

[16] Falcone M., Ferretti R., 2002. Semi-Lagrangian schemes for Hamilton–Jacobi equations, discr-ete representation formulae and Godunov methods. J. Comput. Phys. 175, 559–575.

[17] Farouq N. El., 2017. Deterministic impulse control problems: Two discrete approximations of the quasi-variational inequality, J. Compu. Appl. Math, 309, 200-218.

[18] Ferretti R., Zidani H., 2015. Monotone Numerical Schemes and Feedback Construction for Hybrid Control Systems, J. Optim. Theory Appl., 165, 507–531.

[19] Gonzalez R., Rofman E., 1985. On deterministic control problems: An approximation procedure for the optimal cost II. The nonstationary case, SIAM J. Control Optim. 23, 267–285.

[20] Gonzalez R., Rofman E., 1985. On deterministic control problems: An approximation procedure for the optimal cost I. The stationary problem, SIAM J. Control Optim. 23, 242–266.

[21] Guo B-Z., Wu T-T., 2010. Approximation of optimal feedback control, J. Glob. Optim, 46, 395-422.

[22] Guo B-Z., Wu T-T., 2017. Numerical solution to optimal feedback control by dynamic programming approach, J. Syst. Sci. Complex,

30, 782-802.

[23] Guo B-Z., Sun B., 2005. Numerical solution to the optimal birth feedback control of a population dynamics: viscosity solution approach. Optim. Control Appl. Meth. 26, 229–254.

[24] Kocan M., Soravia P., 1998. A viscosity approach to infinite dimensional Hamilton–Jacobi equations arising in optimal control with state constraints. SIAM J. Control Optim. 36, 1348–1375.

[25] Ramaswamy M., Dharmatti S., 2006. Uniqueness of unbounded viscosity solutions for impulse control problem, J. Math. Anal. Appl., 315, 686–710.

[26] Sassi A., 2017. Numerical schemes for hybrid control systems and chance-constrained optimization problems, 180, universite paris-saclay.

[27] Souganidis P.E., 1985. Approximation schemes for viscosity solutions of Hamilton–Jacobi equations, J. Differential Equations 59, 1–43.

[28] Sun B., Guo B-Z., 2015. Convergence of an upwind finite-difference scheme for Hamilton–Jacobi–Bellman equation in optimal control, IEEE TRANSACTIONS ON AUTOMATIC CONTROL, 60, (11) 3012-3017.