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Bounded Rationality by Temptation in Investment

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Abstract— *Rationality is a common assumption that can be found everywhere in traditional economic theory. We always assume that investors are rational and the preference of investors is consistent. But we can always see irrational behaviors in capital market. We use changing utility function to describe character of time-inconsistency for some people so as to study their investment preference about return and risk. And we explain why there exists so many irrational behaviors in the financial market. Investors themselves suffer from this kind of behaviors and financial institute which is selling risky asset take this opportunity to exploit investors.*

Keywords— ***Bounded Rationality, Time-Inconsistency, Sophisticated, Naïve, Pricing Strategy, Exploitation.***

I. INTRODUCTION

In traditional economic theory, we always assume that investors are rational and the preference of investors is consistent. But we can always see irrational behaviors in capital market. That is because some investors can not be enough knowledgeable to rightly figure out what combination of return and risk they really need. Therefore, there are so many irrational behaviors in the financial market. Investors themselves suffer from this kind of behaviors and financial institute which is selling financial asset take this opportunity to exploit investors.

In reality, not every investor can accurately estimate their extend of risk aversion. We can always see that people are attracted to some higher risky asset beyond their risk-taking ability. For example, people who invest in mature company of large scalar shares with low return and risk may be lured to buy some small growth company shares in order to chase high return later. This changing will make investors to take more risk. If they are rational, they wound not accept it just like their first option. But influenced by gambler psychology they are inclined to amplify their return and ignore the risk. This kind of time-inconsistency can be used by financial institute to exploit a higher consumer surplus and earn a higher profit. Financial institute can provide two kinds of menu. One of them is only low risk asset. Another includes both high risk and low risk asset. People who cannot predict that they may have incentive to take a risk in the future will not refrain themselves from the higher risky asset and choose the menu with more options. After getting stick to the menu, the investors will change to high risk asset by the temptation of financial institute.

There are two kinds of investor, the sophisticated and the naïve. The sophisticated people can perfectly understand their changing in taste. But the naïve people make their decision only depend on their temporary preference. Thus, firm can only earn a normal profit on the sophisticated people but exploit a higher profit from the naïve when the market is monopoly. The multi-selves' approach was originally introduced by Strotz (1956) and developed further by Peleg & Yaari (1973). We consider the two stage of making decision. People firstly choose whether to enter the market, and what kinds of menu they want to choose. Secondly, they will pick one asset in the menu later. Della Vigna & Malmendier (2004) firstly analyze price setting in the presence of dynamically inconsistent preferences. Peleg and Yaari(1973) consider the behaviors of economic agent whose preferences change over time and whether or not he can escape the predicament of yesterday's actions being non-optimal, when viewed from the vantage point of today's preferences. Eliaz K. and Spiegler(2006) study a contract-theoretic model in which the heterogeneity among agent types is of a "cognitive" nature. In their model, the agent has dynamically inconsistent preferences. Manzini and Mariotti(2009) study two boundedly rational procedures in consumer behavior and show that these procedures can be detected by conditions on observable demand data of the same type as standard revealed preference axioms which is similar to the constraint of our model. Airaudo(2020) use experimental and empirical evidence on preference reversal in consumption choices to introduce temptation with self-control preferences into a New Keynesian model to study the effects of forward guidance in monetary policy. And he studies the representative agent' behavior of trading off between temptation for immediate satisfaction and long-run best. Schneider(2020) introduce a model of temptation-biased preferences that generalizes quasi-hyperbolic discounting and quasi-rank-dependent probability weighting. His model explains empirically observed interactions between risk and time preferences and empirically observed correlations between expected utility violations and discounted utility violations. Frederick S., Loewenstein G. & O'Donoghue(2002)review empirical research on intertemporal choice. Della Vigna & Malmendier (2004) analyze the profit-maximizing contract design of firms if consumers have time-inconsistent preferences and are partially naive about it. O'Donoghue T. & Rabin(2015) study present bias that people are susceptible to the over-pursuit of immediate gratification dates (at least) to the ancient Greeks.

In section 2, we describe the basic model and assumption. In section 3, we classify the investors as naïve and sophisticated, solve the pricing choice of the firm from asset to portfolio in different market structure and analyze the possibility of educating the naïve investors by firms. In section 4, we analyze whether the prices and menu set by firm can lead to separating equilibrium. In section 5, we analyze the welfare of stakeholders. In section 6, we give our conclusions.

II. MODEL

The utility function of investor is constant absolutely risk aversion (CARA) utility function, that is

$$u(R) = -e^{-\delta R},$$

where δ is the absolute risk aversion coefficient. The more investor averse the risk, the bigger δ is. And the utility function is more concave. Let the sophisticated investors' and the naïve investors' absolute risk aversion coefficients are $\delta_s, \delta_n \in \{\delta_1, \delta_2\}$, where $\delta_1 > \delta_2 > 0$.

There are two kinds of assets. Assume the return of asset comply with normal distribution. Asset 1 and 2's mean returns are r_1 and r_2 respectively. Asset 1 and 2's variances are σ_1^2 and σ_2^2. We assume that $r_1 < r_2$ and $\sigma_1^2 < \sigma_2^2$. The covariance between asset 1 and asset 2 is σ_{12}. And the cost of asset 1 and 2 for market maker is c_1 and c_2 respectively. Because of marginal return decreases, c_1, c_2 satisfy $r_1 - c_1 > r_2 - c_2$.

And we also assume that when the absolute risk aversion coefficient of investor is δ_1 the most efficient outcome is to choose asset 1 and when the absolute risk aversion coefficient of investor is δ_2 the most efficient outcome is to choose asset 2. That is

$$E\left(-e^{-\delta_1(r_1-c_1)}\right) > \max\left\{E\left(-e^{-\delta_1(r_2-c_2)}\right), E\left(-e^{-\delta_1 r_f}\right)\right\}$$

and

$$E\left(-e^{-\delta_2(r_2-c_2)}\right) > \max\left\{E\left(-e^{-\delta_2(r_1-c_1)}\right), E\left(-e^{-\delta_2 r_f}\right)\right\}$$

which equal to

$$r_1 - c_1 - \frac{1}{2}\delta_1\sigma_1^2 > \max\left\{r_2 - c_2 - \frac{1}{2}\delta_1\sigma_2^2, r_f\right\}$$

and

$$r_2 - c_2 - \frac{1}{2}\delta_2\sigma_2^2 > \max\left\{r_1 - c_1 - \frac{1}{2}\delta_2\sigma_1^2, r_f\right\}$$

according to log normal distribution.

Firms will set two kinds menu for the sophisticated and the naïve respectively.

The investors decide whether to join the market at the first stage. If they do not get into the market, they can invest in risk-free asset and get payoff r_f finally. If they do choose one of the menu, they must choose a portfolio in market maker's menu.

III. EQUILIBRIUM

3.1 The Sophisticated

For the sophisticated people, they know that they are very unpleasant with risk (their absolute risk aversion coefficient is δ_1) but they will be tempted to take more risk at future (their absolute risk aversion coefficient will change to δ_1). They will decide to choose menu which can prohibit themselves from gambling or risk-free asset.

3.2 The Naïve

For the naïve people, they think they are highly risk aversion at stage 1 (their absolute risk aversion coefficient is δ_1). And they will make investment decision according to expected utility function $E(-e^{-\delta_1 R})$. But they do not know that they will be tempted to take more risk at the second stage(their absolute risk aversion coefficient changes to δ_2). Therefore, they will not try to constraint themselves from gambling in the second stage.

3.3 The Firm

In different market structure, there can be different number of market makers. If there is only one firm, he can make price of different menu and investors decide to take or stay out. If there are many firms, investor can choose one of them according to the menu they provide at the first stage. But once they choose a menu, they can only change their portfolio within the menu that they choose at the second stage. Firms can make use of the unknowing of temptation of naïve investors and attract them by safe asset. At the second stage, firms provide a riskier asset instead to earn higher profit. For simplicity, we first study the situation that firm can only provide asset 1 or 2 for one investor. Let the price charged for investor be M. And then, we will consider a more complex situation that firm can make arbitrary portfolio between asset 1 and 2.

3.4 Monopoly with Asset

For the sophisticated investors, the profit maximizing problem is

$$\max M_s^m - c_i \quad (1)$$

$$s.t. E\left(-e^{-\delta_1 (r_i - M_s^m)}\right) \geq E\left(-e^{-\delta_1 r_f}\right) \quad (2)$$

To maximize the objective function, the constraint (2) must bind, we have

$$E\left(-e^{-\delta_1 (R - M_s^m)}\right) = E\left(-e^{-\delta_1 r_f}\right)$$

Because the return complies with normal distribution, the utility function complies with log-normal distribution, that is

$$R - M_s^m - \frac{1}{2}\delta_1 \sigma_i^2 = r_f.$$

Take $M_s^m = R - \frac{1}{2}\delta_1 \sigma_i^2 - r_f$ into the objective function (1), the profit of the firm earned from the sophisticated investors is

$$M_s^m - c = R - \frac{1}{2}\delta_1 \sigma_i^2 - r_f - c \leq r_1 - c_1 - \frac{1}{2}\delta_1 \sigma_1^2 - r_f.$$

Thus, the optimal price set for sophisticated investor is $M_s^m = r_1 - \frac{1}{2}\delta_1 \sigma_1^2 - r_f$. The profit earned from sophisticated investors is $\pi_s^m = r_1 - \frac{1}{2}\delta_1 \sigma_1^2 - r_f - c_1$.

For the naïve investors, the profit maximizing problem is

$$\max M_{n2}^m - c_2$$

$$s.t. E\left(-e^{-\delta_2 (r_2 - M_{n2}^m)}\right) \geq E\left(-e^{-\delta_2 (r_1 - M_{n1}^m)}\right) \quad (3)$$

$$E\left(-e^{-\delta_1 (r_1 - M_{n1}^m)}\right) \geq E\left(-e^{-\delta_1 (r_2 - M_{n2}^m)}\right) \quad (4)$$

$$E\left(-e^{-\delta_1(r_1 - M_{n1}^m)}\right) \geq E\left(-e^{-\delta_1 r_f}\right) \quad (5)$$

The constraints are equal to

$$r_2 - M_{n2}^m - \frac{1}{2}\delta_2\sigma_2^2 \geq r_1 - M_{n1}^m - \frac{1}{2}\delta_2\sigma_1^2 \quad (3')$$

$$r_1 - M_{n1}^m - \frac{1}{2}\delta_1\sigma_1^2 \geq r_2 - M_{n2}^m - \frac{1}{2}\delta_1\sigma_2^2 \quad (4')$$

$$r_1 - M_{n1}^m - \frac{1}{2}\delta_1\sigma_1^2 \geq r_f \quad (5')$$

In order to set a high M_{n2}^m, constraint $(3')$ must bind, otherwise we can increase M_{n2}^m by an arbitrary small $\varepsilon > 0$ to earn higher profit. Besides, constraint $(5')$ must bind, otherwise we can increase both of M_{n1}^m and M_{n2}^m by $\varepsilon > 0$ to earn higher profit. Thus, we have

$$M_{n1}^m = r_1 - \frac{1}{2}\delta_1\sigma_1^2 - r_f$$

$$M_{n2}^m = r_2 - \frac{1}{2}\delta_2\sigma_2^2 + \frac{1}{2}\delta_2\sigma_1^2 - \frac{1}{2}\delta_1\sigma_1^2 - r_f$$

To examine constraint $(5')$, substitute M_{n1}^m and M_{n2}^m into $(5')$

$$Left - Right = -\frac{1}{2}\delta_2\sigma_2^2 + \frac{1}{2}\delta_2\sigma_1^2 - \frac{1}{2}\delta_1\sigma_1^2 + \frac{1}{2}\delta_1\sigma_2^2 = \frac{\delta_1 - \delta_2}{2}(\sigma_2^2 - \sigma_1^2) > 0,$$

satisfy.

The profit earned from naïve investors is $\pi_n^m = r_2 - \frac{1}{2}\delta_2\sigma_2^2 + \frac{1}{2}\delta_2\sigma_1^2 - \frac{1}{2}\delta_1\sigma_1^2 - r_f - c_2$.

Thus, firm will provide two kinds of menu in monopolistic market, that is $\left\{(asset\ 1, r_1 - \frac{1}{2}\delta_1\sigma_1^2 - r_f)\right\}$ and $\left\{\left(asset\ 1, r_1 - \frac{1}{2}\delta_1\sigma_1^2 - r_f\right), \left(asset\ 2, r_2 - \frac{1}{2}\delta_2\sigma_2^2 + \frac{1}{2}\delta_2\sigma_1^2 - \frac{1}{2}\delta_1\sigma_1^2 - r_f\right)\right\}$.

Proposition 1. In monopolistic market, firms can earn higher profit from naïve investors than sophisticated investors through the temptation to naïve investors buy higher risk asset and take more risk to get higher return.

Proof. $\pi_n^m - \pi_s^m = r_2 - \frac{1}{2}\delta_2\sigma_2^2 + \frac{1}{2}\delta_2\sigma_1^2 - c_2 - r_1 + c_1$

$$= \left(r_2 - \frac{1}{2}\delta_2\sigma_2^2 - c_2\right) - \left(r_1 - c_1 - \frac{1}{2}\delta_2\sigma_1^2\right) > 0.$$

3.5 Competition with Asset

For the sophisticated investors, firms will earn zero profit through competition. Therefore, they will set the price of asset 1 at $M_s^c = c_1$.

For the naïve investors, firms will try to make a low M_{n1}^c to attract investors. Rewrite the constraints $(3'), (4'), (5')$ as

$$M_{n1}^c \geq r_1 - \frac{1}{2}\delta_2\sigma_1^2 - r_2 + M_{n2}^c + \frac{1}{2}\delta_2\sigma_2^2 \quad (6)$$

$$M_{n1}^c \leq r_1 - \frac{1}{2}\delta_1\sigma_1^2 - r_2 + M_{n2}^c + \frac{1}{2}\delta_1\sigma_2^2 \quad (7)$$

$$M_{n1}^c \leq r_1 - \frac{1}{2}\delta_1\sigma_1^2 - r_f \quad (8)$$

In order to make M_{n1}^c be the lowest, the firms need to make M_{n2}^c as low as possible. Since the profit is non-negative, M_{n2}^c is not less than c_2. Thus, we have $M_{n2}^c = c_2$ and $M_{n1}^c = r_1 - \frac{1}{2}\delta_2\sigma_1^2 - r_2 + c_2 + \frac{1}{2}\delta_2\sigma_2^2 < c_1$ through equality of (6). Through examination (7) and (8) is satisfied.

Proposition 2. In competitive market, the Nash equilibrium is that firms choose $M_s^c = c_1$, $M_{n1}^c = r_1 - \frac{1}{2}\delta_2\sigma_1^2 - r_2 + c_2 + \frac{1}{2}\delta_2\sigma_2^2$, $M_{n2}^c = c_2$. And firms can only earn zero profit from naïve investors and sophisticated investors through the temptation to naïve investors to take more risk and get higher return.

Proof. It is a Nash equilibrium because if some firms want to charge M_{n1}^c less to tempt naïve investors, the constraint (6) will be violated. Then at second stage, naïve investors will stick to asset 1 and sophisticated investors will also choose the menu $\{M_{n1}^c, M_{n2}^c\}$ rather than $\{M_s^c\}$. Then the profit earned by firms is

$$M_{n1}^c - c_1 < \left(r_1 - c_1 - \frac{1}{2}\delta_2\sigma_1^2\right) - \left(r_2 - c_2 - \frac{1}{2}\delta_2\sigma_2^2\right) < 0.$$

Thus, there is no opportunity to increase profit.

3.6 Education with Asset

If the market is monopolistic, firm can earn much profit from naïve investors than sophisticated investors. Thus, there is no incentive for firm to educate naïve investors.

If the market is competitive, the outcome depends. If there are only naïve investors, firms can educate naïve investors and provide asset 1 with the price $c_1 + \varepsilon$ and earn a positive profit ε, because they do not need to provide menu $\{M_s^c\}$ without sophisticated investors in the market.

3.7 Monopoly with Portfolio

In monopolistic market, firm can make arbitrary portion of portfolio with asset 1 and 2. For sophisticated investors, the profit maximizing problem is

$$\max M_{sp}^m - \theta_s^m c_1 - (1-\theta_s^m)c_2$$

$$s.t.\, E\left(-e^{-\delta_1(r_{sp}^m - M_{sp}^m)}\right) \geq E\left(-e^{-\delta_1 r_f}\right) \quad (9)$$

where $r_{sp}^m = \theta_s^m r_1 + (1-\theta_s^m) r_2$.

At the optimum, (9) must bind. That is $M_{sp}^m = \theta_s^m r_1 + (1-\theta_s^m)r_2 - \frac{1}{2}\delta_1({\theta_s^m}^2\sigma_1^2 + (1-\theta_s^m)^2 + 2\theta_s^m(1-\theta_s^m)\sigma_{12}) - r_f$.

Then the optimal problem becomes

$$\max\ \theta_s^m r_1 + (1-\theta_s^m)r_2 - \frac{1}{2}\delta_1({\theta_s^m}^2\sigma_1^2 + (1-\theta_s^m)^2\sigma_2^2$$

$$+2\theta_s^m(1-\theta_s^m)\sigma_{12} - r_f - \theta_s^m c_1 - (1-\theta_s^m)c_2$$

$$F.O.C\quad r_1 - r_2 - \delta_1\sigma_1^2\theta_s^m + \delta_1\sigma_2^2(1-\theta_s^m) - \delta_1\sigma_{12}(1-2\theta_s^m) - c_1 + c_2 = 0$$

We have $\theta_s^m = \frac{r_1 - r_2 - c_1 + c_2 + \delta_1(\sigma_2^2 - \sigma_{12})}{\delta_1(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})}$.

For naïve investors, the profit maximizing problem is

$$\max M_{np2}^m - \theta_{n2}^m c_1 - (1-\theta_{n2}^m)c_2$$

$$s.t.\, E\left(-e^{-\delta_2(r_{np2}^m - M_{np2}^m)}\right) \geq E\left(-e^{-\delta_2(r_{np1}^m - M_{np1}^m)}\right) \quad (10)$$

$$E\left(-e^{-\delta_1(r_{np1}^m - M_{np1}^m)}\right) \geq E\left(-e^{-\delta_1(r_{np2}^m - M_{np2}^m)}\right) \quad (11)$$

$$E\left(-e^{-\delta_1(r_{np1}^m - M_{np1}^m)}\right) \geq E\left(-e^{-\delta_1 r_f}\right) \quad (12)$$

Constraints (10), (11), (12) can be rewrite as

$$\theta_{np2}^m r_1 + \left(1-\theta_{np2}^m\right)r_2 - M_{np2}^m - \frac{1}{2}\delta_2\left({\theta_{np2}^m}^2\sigma_1^2 + \left(1-\theta_{np2}^m\right)^2\sigma_2^2 + 2\theta_{np2}^m\left(1-\theta_{np2}^m\right)\sigma_{12}\right)$$

$$\geq \theta_{np1}^{m} r_1 + (1-\theta_{np1}^{m}) r_2 - M_{np1}^{m} - \frac{1}{2}\delta_2 \left({\theta_{np1}^{m}}^2 \sigma_1^2 + (1-\theta_{np1}^{m})^2 \sigma_2^2 + 2\theta_{np1}^{m}(1-\theta_{np1}^{m})\sigma_{12} \right) \quad (10')$$

$$\theta_{np1}^{m} r_1 + (1-\theta_{np1}^{m}) r_2 - M_{np1}^{m} - \frac{1}{2}\delta_1 \left({\theta_{np1}^{m}}^2 \sigma_1^2 + (1-\theta_{np1}^{m})^2 \sigma_2^2 + 2\theta_{np1}^{m}(1-\theta_{np1}^{m})\sigma_{12} \right)$$

$$\geq \theta_{np2}^{m} r_1 + (1-\theta_{np2}^{m}) r_2 - M_{np2}^{m} - \frac{1}{2}\delta_1 \left({\theta_{np2}^{m}}^2 \sigma_1^2 + (1-\theta_{np2}^{m})^2 \sigma_2^2 + 2\theta_{np2}^{m}(1-\theta_{np2}^{m})\sigma_{12} \right) \quad (11')$$

$$\theta_{np1}^{m} r_1 + (1-\theta_{np1}^{m}) r_2 - M_{np1}^{m} - \frac{1}{2}\delta_1 \left({\theta_{np1}^{m}}^2 \sigma_1^2 + (1-\theta_{np1}^{m})^2 \sigma_2^2 + 2\theta_{np1}^{m}(1-\theta_{np1}^{m})\sigma_{12} \right) \geq r_f \quad (12')$$

At the optimum, $(10')$ must bind. Otherwise firm can increase M_{np2}^{m} by an arbitrary small positive ε to earn a higher profit. And $(12')$ must bind too, otherwise firm can increase both M_{np1}^{m} and M_{np2}^{m} by an arbitrary small positive ε to earn a higher profit. Thus, we have

$$M_{np1}^{m} = \theta_{np1}^{m} r_1 + (1-\theta_{np1}^{m}) r_2 - \frac{1}{2}\delta_1 \left({\theta_{np1}^{m}}^2 \sigma_1^2 + (1-\theta_{np1}^{m})^2 \sigma_2^2 + 2\theta_{np1}^{m}(1-\theta_{np1}^{m})\sigma_{12} \right) - r_f$$

and

$$M_{np2}^{m} = \theta_{np2}^{m} r_1 + (1-\theta_{np2}^{m}) r_2 - \frac{1}{2}\delta_2 \left({\theta_{np2}^{m}}^2 \sigma_1^2 + (1-\theta_{np2}^{m})^2 \sigma_2^2 + 2\theta_{np2}^{m}(1-\theta_{np2}^{m})\sigma_{12} \right)$$
$$+ \frac{1}{2}\delta_2 \left({\theta_{np1}^{m}}^2 \sigma_1^2 + (1-\theta_{np1}^{m})^2 \sigma_2^2 + 2\theta_{np1}^{m}(1-\theta_{np1}^{m})\sigma_{12} \right)$$
$$- \frac{1}{2}\delta_1 \left({\theta_{np1}^{m}}^2 \sigma_1^2 + (1-\theta_{np1}^{m})^2 \sigma_2^2 + 2\theta_{np1}^{m}(1-\theta_{np1}^{m})\sigma_{12} \right) - r_f$$

The profit maximizing problem becomes

$$\max_{\theta_{np1}^{m}, \theta_{np2}^{m}} M_{np2}^{m} - \theta_{np2}^{m} c_1 - (1-\theta_{np2}^{m}) c_2$$

$$\text{F.O.C}_{\theta_{np1}^{m}}\ \delta_2 \sigma_1^2 \theta_{np1}^{m} - \delta_2 \sigma_2^2 (1-\theta_{np1}^{m}) + \delta_2 \sigma_{12}(1-2\theta_{np1}^{m})$$
$$-\delta_1 \sigma_1^2 \theta_{np1}^{m} + \delta_1 \sigma_2^2 (1-\theta_{np1}^{m}) - \delta_1 \sigma_{12}(1-2\theta_{np1}^{m}) = 0$$

We have $\theta_{np1}^{m} = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$, which has the lowest risk for any portfolio asset 1 and 2 constituting.

$$\text{F.O.C}_{\theta_{np2}^{m}}\ r_1 - r_2 - \delta_2 \sigma_1^2 \theta_{np2}^{m} + \delta_2 \sigma_2^2 (1-\theta_{np2}^{m}) - \delta_2 \sigma_{12}(1-2\theta_{np2}^{m}) - c_1 + c_2 = 0$$

We have $\theta_{np2}^{m} = \frac{r_1 - r_2 - c_1 + c_2 + \delta_2(\sigma_2^2 - \sigma_{12})}{\delta_2(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})}$. Portfolio 2 is riskier than portfolio 1 because $0 << \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} < \frac{r_1 - r_2 - c_1 + c_2 + \delta_2(\sigma_2^2 - \sigma_{12})}{\delta_2(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})} < 1$.

Substitute $\theta_{np1}^{m}, \theta_{np2}^{m}$ into $(11')$, we find $(11')$ satisfied.

Proposition 3. In monopolistic market, firms can earn higher profit from naïve investors than sophisticated investors through the temptation to naïve investors buy higher risk portfolio and take more risk to get higher return.

Proof. The difference between profit earned from naïve and sophisticated investors is

$$M_{np2}^{m} - \theta_{np2}^{m} c_1 - (1-\theta_{np2}^{m}) c_2 - M_{sp}^{m} + \theta_s^{m} c_1 + (1-\theta_s^{m}) c_2$$
$$= (\theta_{np2}^{m} - \theta_s^{m})(r_1 - c_1 - r_2 + c_2)$$

$$+\frac{\delta_2-\delta_1}{2}\left[\left({\theta^m_{np1}}^2\sigma_1^2+\left(1-\theta^m_{np1}\right)^2\sigma_2^2+2\theta^m_{np1}\left(1-\theta^m_{np1}\right)\sigma_{12}\right)-\left({\theta^m_{np2}}^2\sigma_1^2+\left(1-\theta^m_{np2}\right)^2\sigma_2^2+2\theta^m_{np2}\left(1-\theta^m_{np2}\right)\sigma_{12}\right)\right]$$
$$>0.$$

3.8 Competition with Portfolio

In competitive market, firms earn zero profit from sophisticated investors. They will provide portfolio as same as they provide for sophisticated investors in monopolistic market $\theta_s^c=\frac{r_1-r_2-c_1+c_2+\delta_1(\sigma_2^2-\sigma_{12})}{\delta_1(\sigma_1^2+\sigma_2^2-2\sigma_{12})}$. The price will equal to the cost of portfolio, that is $M_s^c=\theta_s^c c_1+(1-\theta_s^c)c_2$.

As for naïve investors, firms will try to attract them by making M^m_{np1} as low as possible. In order to make M^m_{np1} minimal, firms will make M^m_{np2} as low as possible. But naïve investors will finally choose M^m_{np2}, so M^m_{np2} must be greater than its cost $\theta^m_{np2}c_1+\left(1-\theta^m_{np2}\right)c_2$ to prevent loss.

Therefore, for the naïve investors, firms must set

$$M^c_{np2}=\theta^c_{np2}c_1+\left(1-\theta^c_{np2}\right)c_2$$

$$M^c_{np1}=\theta^c_{np1}r_1+\left(1-\theta^c_{np1}\right)r_2-\frac{1}{2}\delta_2\left({\theta^c_{np1}}^2\sigma_1^2+\left(1-\theta^c_{np1}\right)^2\sigma_2^2+2\theta^c_{np1}\left(1-\theta^c_{np1}\right)\sigma_{12}\right)$$
$$-\theta^c_{np2}r_1-\left(1-\theta^c_{np2}\right)r_2+\frac{1}{2}\delta_2\left({\theta^c_{np2}}^2\sigma_1^2+\left(1-\theta^c_{np2}\right)^2\sigma_2^2+2\theta^c_{np2}\left(1-\theta^c_{np2}\right)\sigma_{12}\right)$$
$$+\theta^c_{np2}c_1+\left(1-\theta^c_{np2}\right)c_2$$

At the first stage, naïve investors try to maximize their expected utility.

$$\max E\left(-e^{-\delta_1\left(r^c_{np1}-M^c_{np1}\right)}\right)$$

Through first order condition with respect to θ^c_{np1}, we have $\theta^c_{np1}=\frac{\sigma_2^2-\sigma_{12}}{\sigma_1^2+\sigma_2^2-2\sigma_{12}}$. And first order condition with respect to θ^c_{np2} says $\theta^c_{np2}=\frac{r_1-r_2-c_1+c_2+\delta_2(\sigma_2^2-\sigma_{12})}{\delta_2(\sigma_1^2+\sigma_2^2-2\sigma_{12})}$. And

$$\theta^c_{np1}r_1+\left(1-\theta^c_{np1}\right)r_2-M^c_{np1}-\frac{1}{2}\delta_1\left({\theta^c_{np1}}^2\sigma_1^2+\left(1-\theta^c_{np1}\right)^2\sigma_2^2+2\theta^c_{np1}\left(1-\theta^c_{np1}\right)\sigma_{12}\right)$$
$$\geq\theta^c_{np2}r_1+\left(1-\theta^c_{np2}\right)r_2-M^c_{np2}-\frac{1}{2}\delta_1\left({\theta^c_{np2}}^2\sigma_1^2+\left(1-\theta^c_{np2}\right)^2\sigma_2^2+2\theta^c_{np2}\left(1-\theta^c_{np2}\right)\sigma_{12}\right)$$
$$\theta^c_{np1}r_1+\left(1-\theta^c_{np1}\right)r_2-M^c_{np1}-\frac{1}{2}\delta_1\left({\theta^c_{np1}}^2\sigma_1^2+\left(1-\theta^c_{np1}\right)^2\sigma_2^2+2\theta^c_{np1}\left(1-\theta^c_{np1}\right)\sigma_{12}\right)$$
$$\geq r_f$$

are satisfied.

Proposition 4. In competitive market, the Nash equilibrium is that firms choose $M_s^m=\theta_s^c c_1+(1-\theta_s^c)c_2$, where $\theta_s^c=\frac{r_1-r_2-c_1+c_2+\delta_1(\sigma_2^2-\sigma_{12})}{\delta_1(\sigma_1^2+\sigma_2^2-2\sigma_{12})}$. And

$$M^c_{np2}=\theta^c_{np2}c_1+\left(1-\theta^c_{np2}\right)c_2,$$

$$M^c_{np1}=\theta^c_{np1}r_1+\left(1-\theta^c_{np1}\right)r_2-\frac{1}{2}\delta_2\left({\theta^c_{np1}}^2\sigma_1^2+\left(1-\theta^c_{np1}\right)^2\sigma_2^2+2\theta^c_{np1}\left(1-\theta^c_{np1}\right)\sigma_{12}\right)$$
$$-\theta^c_{np2}r_1-\left(1-\theta^c_{np2}\right)r_2+\frac{1}{2}\delta_2\left({\theta^c_{np2}}^2\sigma_1^2+\left(1-\theta^c_{np2}\right)^2\sigma_2^2+2\theta^c_{np2}\left(1-\theta^c_{np2}\right)\sigma_{12}\right)$$

$+\theta^c_{np2}c_1+\left(1-\theta^c_{np2}\right)c_2$, where $\theta^c_{np1}=\frac{\sigma_2^2-\sigma_{12}}{\sigma_1^2+\sigma_2^2-2\sigma_{12}}$ and $\theta^c_{np2}=\frac{r_1-r_2-c_1+c_2+\delta_2(\sigma_2^2-\sigma_{12})}{\delta_2(\sigma_1^2+\sigma_2^2-2\sigma_{12})}$. Firms can only earn zero profit from naïve investors and sophisticated investors through the temptation to naïve investors to buy a more risk portfolio.

3.9 Education with Portfolio

In monopolistic market, firms can earn higher profit from naïve investors. Thus, firms will not educate naïve investors.

In competitive market, it depends. If there are only naïve investors, firm can provide menu $\{(\theta_s^c, M_s^c + \varepsilon)\}$, to earn a positive profit ε.

4.Separating Equilibrium

In monopolistic market with asset, firms provide menus $\left\{(asset\ 1, r_1 - \frac{1}{2}\delta_1\sigma_1^2 - r_f)\right\}$ and $\left\{\left(asset\ 1, r_1 - \frac{1}{2}\delta_1\sigma_1^2 - r_f\right), \left(asset\ 2, r_2 - \frac{1}{2}\delta_2\sigma_2^2 + \frac{1}{2}\delta_2\sigma_1^2 - \frac{1}{2}\delta_1\sigma_1^2 - r_f\right)\right\}$. For sophisticated investors, they know the time-inconsistency of their preference that they will be lure to take more risk. So, they will constraint their action in second stage. So sophisticated investors will choose $\left\{(asset\ 1, r_1 - \frac{1}{2}\delta_1\sigma_1^2 - r_f)\right\}$. For naïve investors, they are not aware of this time-inconsistency, so more options may be better for them. Thus, naïve investors will choose $\left\{\left(asset\ 1, r_1 - \frac{1}{2}\delta_1\sigma_1^2 - r_f\right), \left(asset\ 2, r_2 - \frac{1}{2}\delta_2\sigma_2^2 + \frac{1}{2}\delta_2\sigma_1^2 - \frac{1}{2}\delta_1\sigma_1^2 - r_f\right)\right\}$.

In competitive market with asset, firms provide menus $\{(asset\ 1, c_1)\}$ and $\left\{\left(asset\ 1, r_1 - \frac{1}{2}\delta_2\sigma_1^2 - r_2 + c_2 + \frac{1}{2}\delta_2\sigma_2^2\right), (asset\ 2, c_2)\right\}$. From the assumption, we know $r_1 - \frac{1}{2}\delta_2\sigma_1^2 - r_2 + c_2 + \frac{1}{2}\delta_2\sigma_2^2 < c_1$. Firms will set the price of asset 1 below the cost in second menu to lure the investors choose it. However, the sophisticated investors can find this trick. So, they will choose menu 1. For naïve investors, they will choose menu 2 because asset 1 is cheaper.

In monopolistic market with portfolio, firms provide menu $\left\{(\theta_s^m, \theta_s^m r_1 + (1-\theta_s^m)r_2 - \frac{1}{2}\delta_1(\theta_s^{m2}\sigma_1^2 + (1-\theta_s^m)^2 + 2\theta_s^m(1-\theta_s^m)\sigma_{12}) - r_f)\right\}$ and

$\left\{\left(\theta_{np1}^m, \theta_{np1}^m r_1 + (1-\theta_{np1}^m)r_2 - \frac{1}{2}\delta_1\left({\theta_{np1}^m}^2\sigma_1^2 + (1-\theta_{np1}^m)^2\sigma_2^2 + 2\theta_{np1}^m(1-\theta_{np1}^m)\sigma_{12}\right) - r_f\right), \left(\theta_{np2}^m, \theta_{np2}^m r_1 + (1-\theta_{np2}^m)r_2 - \frac{1}{2}\delta_2\left({\theta_{np2}^m}^2\sigma_1^2 + (1-\theta_{np2}^m)^2\sigma_2^2 + 2\theta_{np2}^m(1-\theta_{np2}^m)\sigma_{12}\right) + \frac{1}{2}\delta_2\left({\theta_{np1}^m}^2\sigma_1^2 + (1-\theta_{np1}^m)^2\sigma_2^2 + 2\theta_{np1}^m(1-\theta_{np1}^m)\sigma_{12}\right)\right)\right\}$. At the first stage, $(\theta_s^m, \theta_s^m r_1 + (1-\theta_s^m)r_2 - \frac{1}{2}\delta_1(\theta_s^{m2}\sigma_1^2 + (1-\theta_s^m)^2 + 2\theta_s^m(1-\theta_s^m)\sigma_{12}) - r_f)$ and $\left(\theta_{np1}^m, \theta_{np1}^m r_1 + (1-\theta_{np1}^m)r_2 - \frac{1}{2}\delta_1\left({\theta_{np1}^m}^2\sigma_1^2 + (1-\theta_{np1}^m)^2\sigma_2^2 + 2\theta_{np1}^m(1-\theta_{np1}^m)\sigma_{12}\right) - r_f\right)$ will give same utility $E(-e^{-\delta_1 r_f})$. It is same as monopolistic market with asset, sophisticated investors will choose menu 1 and naïve investors will choose menu 2.

In competitive market with portfolio, firms provide menu $\{(\theta_s^c, \theta_s^c c_1 + (1-\theta_s^c)c_2)\}$ and

$\left\{\left(\theta_{np1}^c, \theta_{np1}^c r_1 + (1-\theta_{np1}^c)r_2 - \frac{1}{2}\delta_2\left({\theta_{np1}^c}^2\sigma_1^2 + (1-\theta_{np1}^c)^2\sigma_2^2 + 2\theta_{np1}^c(1-\theta_{np1}^c)\sigma_{12}\right) - \theta_{np2}^c r_1 - (1-\theta_{np2}^c)r_2 + \frac{1}{2}\delta_2\left({\theta_{np2}^c}^2\sigma_1^2 + (1-\theta_{np2}^c)^2\sigma_2^2 + 2\theta_{np2}^c(1-\theta_{np2}^c)\sigma_{12}\right) + \theta_{np2}^c c_1 + (1-\theta_{np2}^c)c_2\right), \left(\theta_{np2}^c, \theta_{np2}^c c_1 + (1-\theta_{np2}^c)c_2\right)\right\}$. This is also a separating equilibrium because $M_{np1}^c = \theta_s^c c_1 + (1-\theta_s^c)c_2, M_{np1}^c = \theta_{np2}^c c_1 + (1-\theta_{np2}^c)c_2$ satisfies $(10'), (11'), (12')$. Thus, we must have $E\left(-e^{-\delta_1(r_{sp}^c - M_{sp}^c)}\right) \le E(-e^{-\delta_1(r_{np1}^c - M_{np1}^c)})$.

IV. WELFARE ANALYSIS

In our model, we use different utility function to describe the time-inconsistency. If we want to measure the welfare of investors, we must choose one of the utility functions. That depends on when the investors are rational at making decision. We have assumed that sophisticated investors are rational, their actions can be thought as being full knowledge of their own welfare. Therefore, we could also measure the utility relative to sophisticated investors' behavior.

In the model of temptation to gamble, we use first stage utility function as the real function for investors. That is because we assume that investors will be tempted to take a risk at the second stage. This irrational force will drive investors away them from their rational decision in first stage. Thus, we use a more risk-aversion utility to measure the real welfare of investors. In monopolistic market, firms use this time-consistency to attract investors by providing a cheaper and safer asset even actually selling can cause a loss. When they are bind with the menu, firms will tempt them to change to another option in the menu at the second stage. Through this, firms earn higher profit from naïve investors than sophisticated investors. In this model, every outcome is inefficient for naïve investors because naïve investors always be tempted to change to a higher risk investment strategy whatever market is monopolistic or competitive.

V. CONCLUSION

We analysis one kind of time-inconsistency in investment. Investors' invest choice may change during decision process because there exists temptation to earn higher profit and take more risk. However, this additional return and risk is not helpful for investors. Depends on whether investors can be fully aware of this time-inconsistency, we distinguish investors with sophisticated investors and naïve investors. The sophisticated investors can predict that they will be lured to take more risk in the future. Therefore, they negotiate with the firms to constrain their choice in the future by accepting a menu of only low risk investment choice. In this way, they will not be concerned with the temptation of the firms. But as for the naïve investors, they cannot correctly predict the changing of their risk aversion. It leads that naïve investors choose a more various menu. Firms who are with full knowledge of this time-inconsistency put another option in this menu in order to lure naïve investors to change their investment decision. Through this, firms can earn more profit from naïve investors.

It is an inefficient outcome, because naïve investors finally change the investment choice in the menu. They do not get the most suitable investment choice. All investors should choose the investment choice with lower risk in the social optimization. Even in the competitive market, the outcome is still inefficient. But should government interrupt and try to educate the naïve investors? The answer depends, if firm is in monopolistic market, firm has no incentive to educated the naïve investors because firm can earn more profit from them. But in competitive market, firms can earn interim positive profit through educating the investors if all investors are naïve.

The model do not require market to distinguish investors, investors will automatically choose the menu designed for them.

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