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The Bound of Reachable Sets Estimation for Neutral Markovian Jump Systems with Disturbances

Changchun Shen, Guanke Ma, Jinlin Yang

School of Data Science and Information Engineering, Guizhou Minzu University, Guiyang, China

***Abstract*— The problem of finding an bound of reachable sets for neutral markovian jump systems with bounded peak disturbances is studied in this paper. Up to now, the result related to the bound of reachable sets was rarely proposed for neutral markovian jump systems. Based on the modified Lyapunov-Krasovskii type functional and linear matrix inequality technology, we obtain some delay-dependent results expressed in the form of matrix inequalities containing only one non-convex scalar. Numerical examples are given to illustrate our theoretical results.*

***Keywords*—Reachable set, linear neutral system, Lyapunov-Krasovskii, Linear matrix inequalities.**

I. INTRODUCTION

In practice and engineering applications, many dynamical systems may cause abrupt variations in their structure, due to changes in the interconnections of subsystems, sudden environment changes, and so on. Markovian jump systems, which are modeled by a set of subsystems with transitions among the models determined by a Markov chain taking values in a finite set, have been extensively studied, see [1-5] and the references therein. On other hand, neutral system that the time derivative of state depends not only on the delayed state but also on its time derivative exists in many dynamic systems. Since many complex systems can be modeled preferably as neutral systems, a great deal of attention has been drawn to the study of neutral systems for the last two decades, such as [6-8], and the references therein. Recently, many researchers have paid a lot of attentions on the problem of the dynamic properties for neutral Markov systems [9-12].

The reachable set estimation of dynamic systems is defined as the bounded set containing all reachable state trajectories starting from the origin under the constraint of admissible bounded inputs. Recently, the reachable set estimation problem has received growing attention, which due to its importance in solving the problem of state estimation and parameter estimation. Then, an increasing number of researchers have devoted their efforts to the problem of reachable set estimation [14-23]. In [16], a delay-dependent condition for an ellipsoid bound, which contained five non-convex scalar parameters to be treated as tuning parameters for finding the smallest possible ellipsoid, was derived based on the Razumikhin approach and the S-procedure Kim[18] proposed an improved ellipsoidal bound of the reachable set of reachable states based on constructing a modified the Lyapunov functional and Linear Matrix Inequality Method[13]. This limitation may restrict the scope of application for this method. By using a maximal Lyapunov-Krasovskii functional which was constructed by taking pointwise maximum over a family of Lyapunov functionals, some criteria bounding the reachable set bounding for delayed systems subject to both polytopic uncertainties and bounded peak inputs was derived in [20]. In [21], an ellipsoidal bound of reachable sets for neutral systems with bounded peak disturbances was obtained in the form of matrix inequalities containing only one nonconvex scalar based on the modified augmented Lyapunov-Krasovskii type functional. Lin, He, Wu and Liu[20] concerned with the reachable set estimation for Markovian jump neural networks with time-varying delay and bounded peak inputs. By using the Wirtinger-based integral inequality and the extended reciprocally convex matrix inequality, an ellipsoidal description of the reachable set for the considered neural networks is derived. But the result about the bound of reachable set bounding was rarely proposed for neutral markov systems.

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However, the bound of reachable sets for neutral markovian jump systems with bounded peak disturbances has not been investigated, which motivates this paper. In this paper, we consider the problem of finding an bound of reachable sets for neutral markovian jump systems with bounded peak disturbances. Based on the modified Lyapunov-Krasovskii type functional, some delay-dependent results are derived in the form of matrix inequalities containing only one non-convex scalar. Furthermore, a modified matix inequality is used, to remove the limitation on the variation rate of the delay and obtain a "smaller" bound of reachable sets. Numerical examples illustrate the effectiveness of the obtained results.

II. PROBLEM STATEMENT

Consider the following neutral markovian jump systems with disturbances：

$$\begin{cases} \dot{x}(t) - C_{(t,r_t)}\dot{x}(t-\tau) = A_{(t,r_t)}x(t) \\ \qquad + B_{(t,r_t)}x(t-h(t)) + D_{(t,r_t)}w(t), \\ x(t_0+\theta) \equiv 0, \quad \forall\theta \in [-\rho^*, 0], \end{cases} \quad (1)$$

where $x(t) \in \Re^n$ is the state vector, $\tau > 0$ is a constant neutral delay, the discrete delay $h(t)$ is a time-varying function that satisfies

$$0 \le h(t) \le h_M, \dot{h}(t) \le h_d < 1, w^T(t)w(t) \le w_m^2, \quad (2)$$

$\rho^* = \max(\tau, h_M)$, $\{r_t, t \ge 0\}$ is Markovian process taking values on the probability space in a finite state $\wp = \{1,2,\cdots,N\}$ with generator $\Lambda = \{\lambda_{ij}\}(i,j \in \wp)$ and Λ is described as follows

$$P(r_{t+\Delta} = j \mid r_t = i) = \begin{cases} \lambda_{ij}\Delta + o(\Delta), i \ne j, \\ 1 + \lambda_{ii}\Delta + o(\Delta), i = j, \end{cases} \quad (3)$$

where $\Delta > 0$, $\lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta} = 0$, $\lambda_{ij} \ge 0$ $(i \ne j)$ is the transition probability from i to j at time $t \to t+\Delta$, $\lambda_{ij} = -\sum_{j=1, j\ne i}^{N}\lambda_{ij}$. $A_{(t,r_t)}, B_{(t,r_t)}, C_{(t,r_t)}$ and $D_{(t,r_t)}$ are known constant matrices of the Markov process.

Since the state transition probability of the Markovian jump process is considered in this paper is partially known, the transition probability matrix of Markovian jumping process Λ is defined as

$$\Lambda = \begin{bmatrix} \lambda_{11} & ? & \cdots & \lambda_{1N} \\ ? & \lambda_{22} & \cdots & \lambda_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{1N} & ? & \cdots & \lambda_{NN} \end{bmatrix}, \quad (4)$$

where ? represents the unknown transition rate. For notational clarity, $\forall i \in \wp$, the set U^i denotes $U^i = U_k^i \cup U_{uk}^i$ with

$$\begin{aligned} U_k^i &:= \{ j \mid \lambda_{ij} \ \ is \ \ known \ \ for \ \ j \in \wp\}, \\ U_{uk}^i &:= \{ j \mid \lambda_{ij} \ \ is \ \ unknown \ \ for \ \ j \in \wp\}, \end{aligned} \quad (5)$$

moreover, if $U_k^i \ne \phi$, it is further described as $U_k^i = \{k_1^i, k_2^i, \cdots, k_m^i\}$, where m is a non-negation integer with $1 \le m \le N$ and $k_j^i \in Z^+$, $1 \le k_j^i \le N$, $j = 1,2,\cdots,m$ represent the known element of the ith row and jth column in the state transition probability matrix Λ.

For the sake of brevity, $x(t)$ is used to represent the solution of the system under initial conditions $x(t,t_0,x_0)$, and $\{x(t),t\}$ satisfies the initial condition (x_0, r_0). and its weak infinitesimal generator, acting on function V, is defined in [37].

$$LV(x_t,t,i) = \lim_{\Delta\to 0^+}\frac{1}{\Delta}[E(V(x_{t+\Delta}, t+\Delta, r_{t+\Delta})|_{(x_t, r_t = i)} \\ - V(x_t, t, r_t)]. \quad (6)$$

This paper aims to find a reachable set for neutral markovian jump systems (1) based on the Lyapunov–Krasovskii functional approach. We denote the set of reachable states with $w(t)$ that satisfies (2) by

$$\Re_x = \{x(t) \in \Re^n \mid x(t), w(t) \ \ satisfy \ \ (1) and (2)\}. \quad (7)$$

We will bound $\Re_x$ by an ellipsoid of the form

$$\Im(P,1) = \{x(t) \in \Re^n \mid x(t)^T P x(t) \le 1, P > 0\}. \quad (8)$$

For simplicity, there are the following representations:

$$\begin{aligned} &A_i = A_{(t,r_t=i)}, B_i = B_{(t,r_t=i)}, C_i = C_{(t,r_t=i)}, \\ &D_i = D_{(t,r_t=i)}, \ \ P_i = P_{(t,r_t=i)}. \end{aligned}$$

In this paper, the following Lemma and Assumption are needed.

Lemma 1[13]. Let $V(x(0)) = 0$ and $w^T(t)w(t) \le w_m^2$, if

$$\dot{V}(x_t)) + \alpha V(x_t) - \beta w^T(t)w(t) \le 0, \alpha > 0, \beta > 0,$$

then we have $V(x_t)) \le \frac{\beta}{\alpha}w_m^2$ for all $\forall t \ge 0$.

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Lemma 2[13] Suppose $h>0$ and $x(t)\in\Re^n$, for any positive definite matrix W the following inequality holds

$$-h\int_{t-h}^{t}x^T(s)Wx(s)ds\le\begin{bmatrix}x^T(t) & x^T(t-h)\end{bmatrix}\cdot\begin{bmatrix}-W & W\\ W & -W\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-h)\end{bmatrix}. \quad (9)$$

Lemma 3[18]. For any positive-definite matrix $\Phi\in\Re^n$, scalar $\gamma>0$, vector function $w:[0,\gamma]\to\Re^n$ such that the integrations concerned are well defined, then

$$\left(\int_0^\gamma x(s)ds\right)^T\Phi\left(\int_0^\gamma x(s)ds\right)\le\int_0^\gamma x^T(s)\Phi x(s)ds.$$

III. MAIN RESULT

Our aim is to find an ellipsoid set as small as possible to bound the reachable set defined in (3). In this section, based on Lyapunov method and linear matrix inequality techniques, following Theorems are derived.

Theorem 1 Consider the markov neutral system (1) with constraints (2), if there exist symmetric matrices P_{2i}, P_{3i}, $P_{1i}>0$, $W_i>0$ ($i=1,2,\cdots,N$), $Q>0$, $R>0$, $S>0$ and a scalar $\alpha>0$ satisfying the following matrix inequalities for $i=1,2,\cdots,N$,

$$\Phi_i=\begin{bmatrix}\Phi_{i11} & \Phi_{i12} & \Phi_{i13} & P_{2i}^TC_i & P_{2i}^TD_i\\ * & \Phi_{i22} & P_{3i}^TB_i & P_{3i}^TC_i & P_{3i}^TD_i\\ * & * & \Phi_{i33} & 0 & 0\\ * & * & * & -e^{-\alpha\tau}R & 0\\ * & * & * & * & -\frac{\alpha}{w_m}I\end{bmatrix}<0, \quad (10)$$

$$P_{1j}-W_i\le 0,\quad j\in U_{uk}^i,\quad j\ne i, \quad (11)$$

$$P_{1j}-W_i\ge 0,\quad j\in U_{uk}^i,\quad j=i, \quad (12)$$

where

$$\Phi_{i11}=P_{2i}^TA_i+A_i^TP_{2i}+Q-h_Me^{-\alpha h_M}S+\alpha P_{1i}+\sum_{j\in U_k^i}^{N}\lambda_{ij}(P_{1j}-W_i),$$

$$\Phi_{i12}=P_{1i}-P_{2i}^T+A_i^TP_{3i},$$

$$\Phi_{i13}=P_{2i}^TB+h_Me^{-\alpha h_M}P_{3i},$$

$$\Phi_{i22}=h_MS+R-P_3-P_{3i}^T,$$

$$\Phi_{i33}=-(1-h_d)e^{-\alpha h_M}Q-h_Me^{-\alpha h_M}S.$$

Then, the reachable sets of the system (1) having the constraints (2) is bounded by a boundary $\bigcap_{i\in\wp}\Im(P_{1i},1)$, which $\Im(P_{1i},1)(i\in\wp)$ is defined in (8).

Proof: We choose the following Lyapunov-Krasovskii functional candidate as follows:

$$V(x_t,t,r_t)=\sum_{i=1}^{4}V_i(x_t,t,r_t), \quad (13)$$

where

$$V_1(x_t,t,r_t)=x^T(t)P_{1r_t}x(t)=\begin{bmatrix}x^T(t) & x^T(t-h)\end{bmatrix}\cdot\begin{bmatrix}I & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}P_{1r_t} & 0\\ P_{2r_t} & P_{3r_t}\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-h)\end{bmatrix},$$

$$V_2(x_t,t,r_t)=\int_{t-h(t)}^{t}e^{\alpha(s-t)}x^T(s)Qx(s)ds,$$

$$V_3(x_t,t,r_t)=\int_{t-\tau}^{t}e^{\alpha(s-t)}\dot{x}^T(s)R\dot{x}(s)ds,$$

$$V_4(x_t,t,r_t)=\int_{-h_M}^{0}\int_{t+\theta}^{t}e^{\alpha(s-t)}\dot{x}^T(s)S\dot{x}(s)dsd\theta,$$

where $r_i\in\wp$, P_{2i}, P_{3i} $P_{1i}>0$, $W_i>0$ ($i=1,2,\cdots,N$), $Q>0$, $R>0$, $S>0$ and a scalar $\alpha>0$ are solutions of (10).

First, we show that $V(x_t)$ in (11) is a good L-K functional candidate. For $t-h_M\le s\le t$, we have

$$0<e^{-\alpha h_M}\le e^{-\alpha(s-t)}\le 1.$$

. Furthermore, for $t-h_M\le s\le t$, we have

$$\sum_{i=2}^{4}V_i(x_t,t,r_t)\ge 0.$$

Therefore, we get

$$\begin{cases}V(x_t,t,r_t)=\sum_{i=1}^{4}V_i(x_t,t,r_t)\ge x^T(t)P_{1r_t}x(t),\\ V(\theta)=0,\quad when\quad x(\theta)=0,\theta\in[t-\rho^*,t],\end{cases} \quad (14)$$

Then, for given $r_t=i\in\wp$, $P_{1r_t}=P_{1i}$, $P_{2r_t}=P_{2i}$, $P_{3r_t}=P_{3i}$ and the weak infinitesimal operator L of the stochastic process $x(t)$ along the evolution of $V_k(x_t,t,r_t)$ ($k=1,2,\cdots,N$) are given as

$$LV_1(x_t,t,i_t)=2\begin{bmatrix}x^T(t) & x^T(t-h)\end{bmatrix}\begin{bmatrix}P_{1r_t} & P_{2r_t}^T\\ 0 & P_{3r_t}^T\end{bmatrix}\cdot\begin{bmatrix}x(t)\\ 0\end{bmatrix}+x^T(t)\Big[\sum_{j\in U_k^i}^{N}\lambda_{ij}P_{1j}\Big]x(t)$$

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$$= 2\left[x^T(t) \quad x^T(t-h)\right]\begin{bmatrix} P_{1i} & P_{2i}^T \\ 0 & P_{3i}^T \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ \begin{pmatrix} A_i x(t) - \dot{x}(t) + C\dot{x}(t-\tau) \\ + B_i x(t-h(t)) + Dw(t) \end{pmatrix} \end{bmatrix} + x^T(t)[\sum_{j\in U_k^i}^{N} \lambda_{ij} P_{1j}]x(t)$$

$$= x^T(t)[P_{2i}^T A_i + A_i^T P_{2i}]x(t) + 2x^T(t)[P_{1i} - P_{2i}^T + A_i^T P_{3i}]\dot{x}(t) + 2x^T(t)P_{2i}^T B_i x(t-h(t)) + 2x^T(t)P_{2i}^T C_i x(t-\tau) + 2x^T(t)P_{2i}^T D_i w(t) - \dot{x}^T(t)[P_{3i}^T + P_{3i}]B_i\dot{x}(t) + 2\dot{x}^T(t)P_{3i}^T B_i \cdot x(t-h(t)) + 2\dot{x}^T(t)P_{3i}^T C_i\dot{x}(t-\tau) + 2\dot{x}^T(t)P_{3i}^T D_i w(t) + x^T(t)[\sum_{j=1}^{N} \lambda_{ij} P_{1j}]x(t).$$

Taking into account the situation that the information of transition probabilities is not accessible completely, due to $\sum_{j=1}^{N} \lambda_{ij} = 0$ ($i \in \wp$), the following equation hold for arbitrary appropriate matrices $W_i = W_i^T$ are satisfied

$$-x^T(t)\sum_{j=1}^{N} \lambda_{ij} W_i x(t) = 0\,. \qquad (15)$$

It is trivial to obtain the following equality:

$$LV_1(x_t,t,i) = x^T(t)[P_{2i}^T A_i + A_i^T P_{2i}]x(t) + 2x^T(t)[P_{1i} - P_{2i}^T + A_i^T P_{3i}]\dot{x}(t) + 2x^T(t)P_{2i}^T B_i x(t-h(t)) + 2x^T(t)P_{2i}^T C_i x(t-\tau) + 2x^T(t)P_{2i}^T D_i w(t) - \dot{x}^T(t)[P_{3i}^T + P_{3i}]B_i\dot{x}(t) + 2\dot{x}^T(t)P_{3i}^T B_i \cdot x(t-h(t)) + 2\dot{x}^T(t)P_{3i}^T C_i x(t-\tau) + 2\dot{x}^T(t)P_{3i}^T \cdot D_i w(t) + x^T(t)[\sum_{j\in U_k^i} \lambda_{ij}(P_{1j} - W_i)]x(t) + x^T(t)[\sum_{j\in U_{uk}^i}^{N} \lambda_{ij}(P_{1j} - W_i)]x(t).$$

$$LV_2(x_t,t,i) = x^T(t)Qx(t) - (1-\dot{h}(t))e^{-\alpha h(t)}x^T(t-h(t))Q \cdot x(t-h(t)) - \alpha\int_{t-h(t)}^{t} e^{\alpha(s-t)}x^T(s)Qx(s)ds \le x^T(t)Qx(t) - (1-h_d)e^{-\alpha h_M}x^T(t-h(t))Q \cdot x(t-h(t)) - \alpha V_2,$$

$$LV_3(x_t,t,i) = \dot{x}^T(t)R\dot{x}(t) - e^{-\alpha\tau}\dot{x}^T(t-\tau)R \cdot \dot{x}(t-\tau) - \alpha\int_{t-\tau}^{t} e^{\alpha(s-t)}\dot{x}^T(s)R\dot{x}(s)ds = \dot{x}^T(t)R\dot{x}(t) - e^{-\alpha\tau}\dot{x}^T(t-\tau)R\dot{x}(t-\tau) - \alpha V_3,$$

$$LV_4(x_t,t,i) = h_M^2\dot{x}^T(t)S\dot{x}(t) - h_M\int_{t-h_M}^{t} e^{\alpha(s-t)}\dot{x}^T(s)S\dot{x}(s)ds \le h_M^2\dot{x}^T(t)S\dot{x}(t) - h_M e^{-\alpha h_M}\begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \cdot \begin{bmatrix} -S & S \\ S & -S \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} - \alpha V_4.$$

We further have

$$LV(x_t,t,i) + \alpha V(x_t,t,i) - \frac{\alpha}{w_m^2}w^T(t)w(t) \le x^T(t)[P_{2i}^T A_i + A_i^T P_{2i} + Q - h_M e^{-\alpha h_M}S + \alpha P_{1i} + \sum_{j\in U_k^i}^{N}\lambda_{ij}(P_{1j} - W_i)]x(t) + 2x^T(t)[P_{1i} - P_{2i}^T + A_i^T P_{3i}]\dot{x}(t) + 2x^T(t)[P_{2i}^T B_i + h_M e^{-\alpha h_M}S]x(t-h(t)) + 2x^T(t)P_{2i}^T C_i x(t-\tau) + 2x^T(t)P_{2i}^T D_i w(t) + \dot{x}^T(t)[\tau h_M^2 S + R - P_{3i}^T - P_{3i}]B_i\dot{x}(t) + 2\dot{x}^T(t)P_{3i}^T \cdot B_i x(t-h(t)) + 2\dot{x}^T(t)P_{3i}^T C_i\dot{x}(t-\tau) + 2\dot{x}^T(t)P_{3i}^T D_i w(t) - x^T(t-h(t))[(1-h_d)e^{-\alpha h_M}Q + h_M e^{-\alpha h_M}S]x(t-h(t)) - e^{-\alpha\tau}\dot{x}^T(t-\tau)R\dot{x}(t-\tau) - \frac{\alpha}{w_m^2}w^T(t)w(t) + x^T(t)[\sum_{j=1}^{N}\lambda_{ij}(P_{1j} - W_i)]x(t)$$

$$= \xi^T(t)\Phi_i\xi(t) + +x^T(t)[\sum_{j=1}^{N}\lambda_{ij}(P_{1j} - W_i)]x(t).$$

where Φ_i is the same as defined in the Theorem 1 and $\xi^T(t) = \left[x^T(t) \quad \dot{x}^T(t) \quad x^T(t-h(t)) \quad \dot{x}^T(t-\tau) \quad w^T(t)\right]$.

Thus, from matrix inequalities (10)-(12), we get

$$LV(x_t) + \alpha V(x_t) - \frac{\alpha}{w_m^2}w^T(t)w(t) \le 0, \qquad (16)$$

which means, by Lemma 1, that $V(x_t,t,r_t) \le 1$, and this results in $V_1(x_t,t,i) = x^T(t)P_{1i}{}^T(t) \le 1$ for $i \in \wp$, since $V_2(x_t,t,i) + V_3(x_t,t,i) + V_4(x_t,t,i) \ge 0$ from (14).

So the reachable set of the system (1) is bounded by ellipsoid $\Im(P_{1i},1)(i \in \wp)$ defined in (8), which implies that the reachable sets of the system (1) having the constraints (2) is bounded by a boundary $\bigcap_{i\in\wp}\Im(P_{1i},1)$. This completes the proof.

Remark 1. It should be noted that the more the unknown elements there are in (4), the lower the maximum of time delay will be in Theorem 1. Actually, if all transition probabilities are unknown, the corresponding system can be viewed as a switched neutral markovian jump system under arbitrary switching. Thus, the

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conditions obtained in Theorem 1 will be thereby cover the results for arbitrary switched neutral markovian jump systems with disturbances. In that case, one can see the bounds of reachable sets in Theorem 1 become seriously conservative, for many constraints. Fortunately, we can use the Lyapunov functional method to analyze the bound of reachable sets for the neutral markovian jump system under the assumption that all transition probabilities are not known.

Following a similar line as in proof of Theorem 1, we can obtain the following Theorem.

Theorem 2. Consider the markov neutral system (1) with all elements known in transition rate matrix (4), if there exist symmetric matrices P_{2i}, P_{3i}, $P_{1i}>0$, $W_i>0$ ($i=1,2,\cdots,N$), $Q>0$, $R>0$, $S>0$ and a scalar $\alpha>0$ satisfying the following matrix inequalities for $i=1,2,\cdots,N$,

$$\Phi_i=\begin{bmatrix}\Phi_{i11} & \Phi_{i12} & \Phi_{i13} & P_{2i}^TC_i & P_{2i}^TD_i\\ * & \Phi_{i22} & P_{3i}^TB_i & P_{3i}^TC_i & P_{3i}^TD_i\\ * & * & \Phi_{i33} & 0 & 0\\ * & * & * & -e^{-\alpha\tau}R & 0\\ * & * & * & * & -\dfrac{\alpha}{w_m}I\end{bmatrix}<0, \quad (17)$$

where

$$\Phi_{i11}=P_{2i}^TA_i+A_i^TP_{2i}+Q-h_Me^{-\alpha h_M}S+\alpha P_{1i}+\sum_{j\in U_k^i}^{N}\lambda_{ij}P_{1j},$$

$$\Phi_{i12}=P_{1i}-P_{2i}^T+A_i^TP_{3i},$$

$$\Phi_{i13}=P_{2i}^TB+h_Me^{-\alpha h_M}P_{3i},$$

$$\Phi_{i22}=h_MS+R-P_3-P_{3i}^T,$$

$$\Phi_{i33}=-(1-h_d)e^{-\alpha h_M}Q-h_Me^{-\alpha h_M}S.$$

Then, the reachable sets of the system (1) having the constraints (2) is bounded by a boundary $\bigcap_{i\in\wp}\Im(P_{1i},1)$, which $\Im(P_{1i},1)(i\in\wp)$ is defined in (8).

For finding the bound of reachable sets for neutral Markovian jump systems with all transition probabilities are not known, which indicate that system (1) is a switched neutral delay systems. One can construct the following Lyapunov functional and obtain the Theorem 3.

$$V_1(x_t,t,r_t)=\begin{bmatrix}x^T(t) & x^T(t-h)\end{bmatrix}\begin{bmatrix}I & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}P_{1r_t} & 0\\ P_{2r_t} & P_{3r_t}\end{bmatrix}$$

$$\cdot\begin{bmatrix}x(t)\\ x(t-h)\end{bmatrix}+\int_{t-h(t)}^{t}e^{\alpha(s-t)}x^T(s)Qx(s)ds$$

$$+\int_{t-\tau}^{t}e^{\alpha(s-t)}\dot{x}^T(s)R\dot{x}(s)+\int_{-h_M}^{0}\int_{t+\theta}^{t}e^{\alpha(s-t)}\dot{x}^T(s)S\dot{x}(s)dsd\theta.$$

Following a similar line as in proof of Theorem 1, we can obtain the following Theorem 3.

Theorem 3. Consider the markov neutral system (1) with all elements unknown in transition rate matrix (4), if there exist symmetric matrices P_{2i}, P_{3i}, $P_{1i}>0$, $W_i>0$ ($i=1,2,\cdots,N$), $Q>0$, $R>0$, $S>0$ and a scalar $\alpha>0$ satisfying the following matrix inequalities for $i=1,2,\cdots,N$,

$$\Phi_i=\begin{bmatrix}\Phi_{i11} & \Phi_{i12} & \Phi_{i13} & P_{2i}^TC_i & P_{2i}^TD_i\\ * & \Phi_{i22} & P_{3i}^TB_i & P_{3i}^TC_i & P_{3i}^TD_i\\ * & * & \Phi_{i33} & 0 & 0\\ * & * & * & -e^{-\alpha\tau}R & 0\\ * & * & * & * & -\dfrac{\alpha}{w_m}I\end{bmatrix}<0, \quad (18)$$

where

$$\Phi_{i11}=P_{2i}^TA_i+A_i^TP_{2i}+Q-h_Me^{-\alpha h_M}S+\alpha P_{1i},$$

$$\Phi_{i12}=P_{1i}-P_{2i}^T+A_i^TP_{3i},$$

$$\Phi_{i13}=P_{2i}^TB+h_Me^{-\alpha h_M}P_{3i},$$

$$\Phi_{i22}=h_MS+R-P_3-P_{3i}^T,$$

$$\Phi_{i33}=-(1-h_d)e^{-\alpha h_M}Q-h_Me^{-\alpha h_M}S.$$

Then, the reachable sets of the system (1) having the constraints (2) is bounded by a boundary $\bigcap_{i\in\wp}\Im(P_{1i},1)$, which $\Im(P_{1i},1)(i\in\wp)$ is defined in (8).

Remark 2. The solution for (10-12), (17) or (18), if it exists, need not be unique. It is well-known ([13]) that the volume of $\Im(P_{1i},1)(i\in\wp)$ defined in (8) is proportional to $\sqrt{\det(P_{1i})}$, so the minimization of $\det(P_{1i})^{-\frac{1}{2}}$ is the same as minimizing the volume of $\Im(P_{1i},1)$. That is, maximize $\det(P_{1i})^{\frac{1}{2}}$ subject to $\delta I<P_{1i}$ which can be equivalent to the following optimization problem:

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$$\min_{\delta_i}\{\min_{j\in\wp}\det(P_{ij})^{-\frac{1}{2}}\}$$
$$s.t.\begin{cases} P_{1i}-\delta_i\geq 0, i\in\wp, \\ (10-12),(17) or (18). \end{cases} \quad (19)$$

Remark 3 The matrix inequalities in Theorem 1 and Theorem 2 contain only one non-convex scalar α (for given h_M and h_d, and these become LMI by fixing the scalar α. The feasibility check of a matrix inequality having only one non-convex scalar parameter is numerically tractable, and a local optimum value of α can be found by fminsearch.m.

IV. EXAMPLE

In this section, a numerical example demonstrates the effectiveness of the mentioned above. Consider the neutral markovian jump systems with three operation modes whose state matrices are listed as follow:

$$\begin{cases} \dot{x}(t)-C_{(t,r_t)}\dot{x}(t-\tau)=A_{(t,r_t)}x(t) \\ \qquad\qquad +B_{(t,r_t)}x(t-h(t)+D_{(t,r_t)}w(t), \\ x(t_0+\theta)\equiv 0, \quad \forall\theta\in[-\rho^*,0], \end{cases} \quad (20)$$

where

$$A_1=\begin{bmatrix}-2 & -1\\ 0 & -2\end{bmatrix},\ A_2=\begin{bmatrix}-3 & 0\\ 0 & -2\end{bmatrix},\ A_3=\begin{bmatrix}-2 & 0\\ -1 & -1.5\end{bmatrix},$$

$$B_1=\begin{bmatrix}-1 & 0\\ -1 & -2\end{bmatrix},\ B_2=\begin{bmatrix}-2 & 0\\ -1 & -1\end{bmatrix},\ B_3=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix},$$

$$C_1=\begin{bmatrix}-1 & 0\\ -1 & -2\end{bmatrix},\ C_2=\begin{bmatrix}-2 & 0\\ -1 & -1\end{bmatrix},\ C_3=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix},$$

$$D_1=\begin{bmatrix}-0.15\\ 0.15\end{bmatrix},\ D_2=\begin{bmatrix}-0.14\\ 0.35\end{bmatrix},\ D_3=\begin{bmatrix}-0.2\\ 0.3\end{bmatrix}.$$

The transition rate matrix Λ is considered as the following three cases.

Case 1. The transition rate matrix Λ is completely known, which is considered as

$$\Lambda=\begin{bmatrix}-0.6 & 0.2 & 0.4\\ 0.6 & -1 & 0.4\\ 0.3 & 0.5 & -0.8\end{bmatrix}.$$

Case 2. The transition rate matrix Λ is partly known, which is considered as

$$\Lambda=\begin{bmatrix}-0.6 & 0.2 & 0.4\\ ? & -1 & ?\\ ? & ? & ?\end{bmatrix}.$$

Case 3. The transition rate matrix Λ is completely unknown, which is considered as

$$\Lambda=\begin{bmatrix}? & ? & ?\\ ? & ? & ?\\ ? & ? & ?\end{bmatrix}.$$

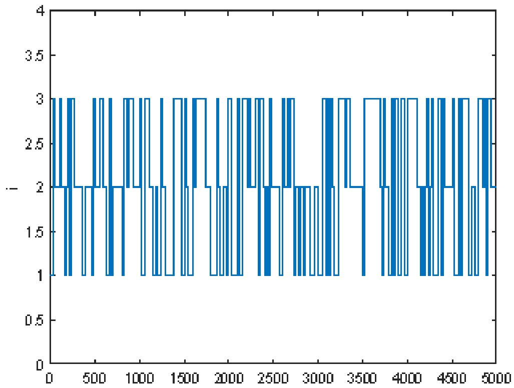

Fig. 1: Random jumping mode r(t) of neutral markov jump system (20)

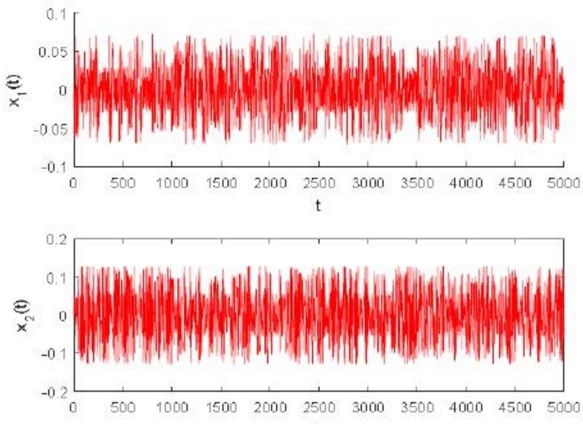

Fig. 2: The time responses of state variable x(t) of neutral markov jump system (20) in case 1

Firstly, by giving the transition probabilities Λ, a possible mode evolution of the neutral markov jump systm (21) is derived as shown in Fig. 1. Based on the mode evolution shown in Fig. 1, and choosing disturbances $w(t)$ as the random signal satisfying $w^T(t)w(t)\leq 1$, all the reachable states of neutral markov jump systm (21) starting from the origin are given in Fig.2.

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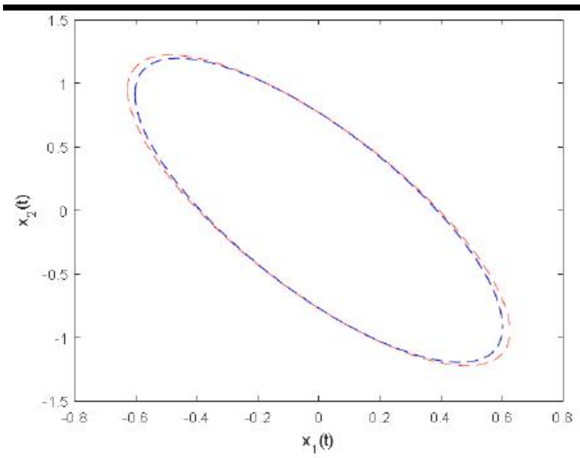

Fig. 3: The bound of reachable *sets for neutral markov jump system (20) in case 2*

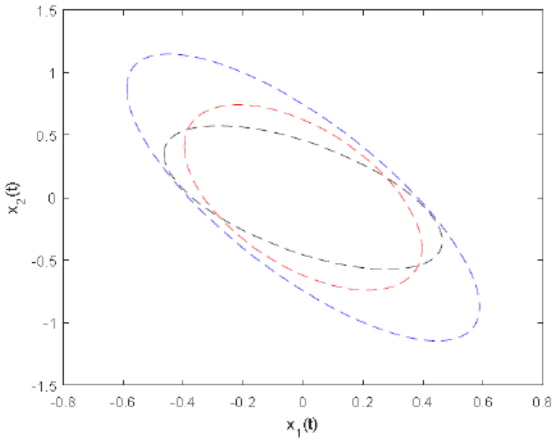

Fig. 4: The bound of reachable sets for neutral markov jump system (20) in case 3

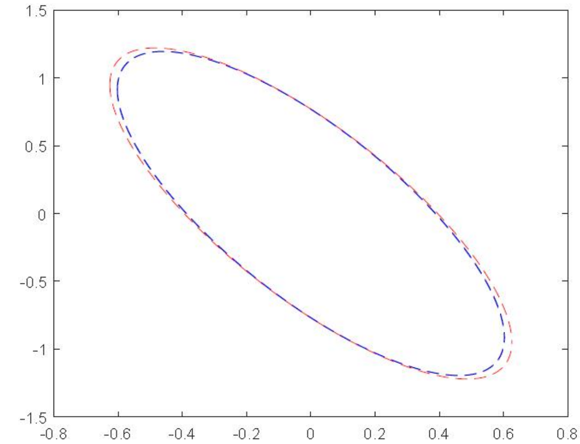

Fig. 5: Random jumping mode r(t) of neutral markov jump system (20).

By using theorem 2 and solving the optimization problem (20) in case 1, we can obtain minimization of $\det(P_{1i})^{-\frac{1}{2}}$ is 0.2119 when $\alpha = 0.1$, and the corresponding feasible matrices are given as $P_{11} = \begin{bmatrix} 7.3166 & 3.5856 \\ 3.5856 & 4.7999 \end{bmatrix}$, $P_{12} = \begin{bmatrix} 9.0948 & 2.6437 \\ 2.6437 & 2.5838 \end{bmatrix}$, $P_{13} = \begin{bmatrix} 6.8632 & 2.6743 \\ 2.6743 & 1.8016 \end{bmatrix}$. The reachable sets of the system (23) in case 1 is bounded by a intersection of three ellipsoids: $\bigcap_{i=1}^{3} \Im(P_{1i}, 1)$, which is depicted in Fig. 3.

By using theorem 1 and solving the optimization problem (20) in case 2, we can obtain the minimization of $\det(P_{1i})^{-\frac{1}{2}}$ is 0.4621 when $\alpha = 0.1$ and the corresponding feasible matrices are given as $P_{11} = \begin{bmatrix} 6.6245 & 2.5552 \\ 2.5552 & 1.6846 \end{bmatrix}$, $P_{12} = \begin{bmatrix} 6.4226 & 2.5544 \\ 2.5544 & 1.6857 \end{bmatrix}$, $P_{13} = \begin{bmatrix} 6.6851 & 2.5788 \\ 2.5788 & 1.6952 \end{bmatrix}$. The reachable sets of the system (23) in case 1 is bounded by a intersection of three ellipsoids: $\bigcap_{i=1}^{3} \Im(P_{1i}, 1)$, which is depicted in Fig.4.

By using theorem 3 and solving the optimization problem (20) in case 3, we can obtain the maximize $\delta = 0.65$ when $\alpha = 0.1$, and the corresponding feasible matrices are given as $P_{11} = \begin{bmatrix} 8.1386 & 4.3002 \\ 4.3002 & 4.8168 \end{bmatrix}$, $P_{12} = \begin{bmatrix} 9.0931 & 2.6563 \\ 2.6563 & 2.5718 \end{bmatrix}$, $P_{13} = \begin{bmatrix} 6.6807 & 2.6735 \\ 2.6735 & 1.8013 \end{bmatrix}$. The reachable sets of the system (23) in case 1 is bounded by a intersection of three ellipsoids: $\bigcap_{i=1}^{3} \Im(P_{1i}, 1)$, which is depicted in Fig.5.

V. CONCLUSION

In This paper, the problem of robust stability for a class of uncertain neutral systems with time-varying delays is investigated. Sufficient conditions are given in terms of linear matrix inequalities which can be easily solved by LMI Toolbox in Matlab. Numerical examples are given to indicate significant improvements over some existing results.

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