

A New Method to Solving Generalized Fuzzy Transportation Problem-Harmonic Mean Method

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Abstract— *Transportation Problem is one of the models in the Linear Programming problem. The objective of this paper is to transport the item from the origin to the destination such that the transport cost should be minimized, and we should minimize the time of transportation. To achieve this, a new approach using harmonic mean method is proposed in this paper. In this proposed method transportation costs are represented by generalized trapezoidal fuzzy numbers. Further comparative studies of the new technique with other existing algorithms are established by means of sample problems.*

Keywords— *Fuzzy Transportation Problem (FTP); Generalized Trapezoidal Fuzzy Number (GTrFN); Ranking function; Harmonic Mean Method (HMM).*

I. INTRODUCTION

In transportation problem, different sources supply to different destinations of demand in such a way that the transportation cost should be minimized. We can obtain basic feasible solution by three methods. They are

1. North West Corner method
2. Least Cost method
3. Vogel's Approximation method (VAM)

In these three methods, VAM method is best according to the literature. We check the optimality of the transportation problem by MODI method. The transportation problem is classified into two types. They are balanced transportation problem and unbalanced transportation problem. If the number of sources is equal to number of demands, then it is called balanced transportation problem. If not, it is called unbalanced transportation problem. If the source of item is greater than the demand, then we should add dummy column to make the problem as balanced one. If the demand is greater than the source, then we should add the dummy row to convert the given unbalanced problem to balanced transportation problem.

Transportation problem is an important network structured in linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in the problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling, and many others. In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way i.e., in crisp environment. However, in many cases the decision makers has no crisp information about the coefficients belonging to the transportation problem. In these cases, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and the fuzzy transportation problem (FTP) appears in natural way. The basic transportation problem was originally developed by Hitchcock [14]. The transportation problems can be modeled as a standard linear programming problem, which can then

be solved by the simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to the transportation problem can be made quite efficient in terms of how to evaluate the necessary simplex method information (Variable to enter the basis, variable to leave the basis and optimality conditions). Charnes and Cooper [4] developed a stepping stone method which provides an alternative way of determining the simplex method information. Dantzig and Thapa [8] used simplex method to the transportation problem as the primal simplex transportation method. An Initial Basic Feasible Solution (IBFS) for the transportation problem can be obtained by using the North- West Corner rule, Row Minima Method, Column Minima Method, Matrix Minima Method or Vogel's Approximation Method (VAM) [21]. The Modified Distribution Method (MODI) [5] is useful for finding the optimal solution of the transportation problem. In general, the transportation problems are solved with the assumptions that the coefficients or cost parameters are specified in a precise way i.e., in crisp environment. In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy number [23] may represent the data. Hence fuzzy decision making method is used here.

Zimmermann [24] showed that solutions obtained by fuzzy linear programming method and are always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Chanas et al. [2] presented a fuzzy linear programming model for solving transportation problems with crisp cost coefficient, fuzzy supply and demand values. Chanas and Kuchta [3] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution. Saad and Abbas [22] discussed the solution algorithm for solving the transportation problem in fuzzy environment. Liu and Kao [18] described a method for solving fuzzy transportation problem based on extension principle. Gani and Razak [13] presented a two stage cost minimizing fuzzy transportation problem (FTP) in which supplies and demands are trapezoidal fuzzy numbers. A parametric approach is used to obtain a fuzzy solution and the aim is to minimize the sum of the transportation costs in

two stages. Lin [166] introduced a genetic algorithm to solve transportation problem with fuzzy objective functions. Dinagar and Palanivel [9] investigated FTP, with the aid of trapezoidal fuzzy numbers. Fuzzy modified distribution method is proposed to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [20] proposed a new algorithm namely, fuzzy zero point method for finding a fuzzy optimal solution for a FTP, where the transportation cost, demand and supply are represented by trapezoidal fuzzy numbers. Edward Samuel [10-12] a solving generalized trapezoidal fuzzy transportation problems, where precise values of the transportation costs only, but there is no uncertain about the demand and supply. In this paper, a proposed method, namely, Harmonic Mean Method (HMM) is used for solving a special type of fuzzy transportation problem by assuming that a decision maker is uncertain about the precise values of transportation costs only. In the proposed method transportation costs are represented by generalized trapezoidal fuzzy numbers. To illustrate the proposed method HMM a numerical example is solved. The proposed method HMM is easy to understand and to apply in real life transportation problems for the decision makers.

II. PRELIMINARIES

In this section, some basic definitions, arithmetic operations and an existing method for comparing generalized fuzzy numbers are presented.

2.1. Definition [15] A fuzzy set \tilde{A} , defined on the universal set of real numbers \mathfrak{R} is said to be fuzzy number if its membership function has the following characteristics:

- (i) $\mu_{\tilde{A}}(x) : \mathfrak{R} \rightarrow [0,1]$ is continuous.
- (ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a) \cup [d, \infty)$.
- (iii) $\mu_{\tilde{A}}(x)$ Strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$
- (iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a < b < c < d$.

2.2. Definition [15] A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x < b, \\ 1 & , b \leq x \leq c, \\ \frac{(x-d)}{(c-d)}, & c < x \leq d, \\ 0 & , \text{otherwise.} \end{cases}$$

2.3. Definition [6] A fuzzy set \tilde{A} , defined on the universal set of real numbers \mathfrak{R} , is said to be generalized fuzzy number if its membership function has the following characteristics:

- (i) $\mu_{\tilde{A}}(x) : \mathfrak{R} \rightarrow [0, \omega]$ is continuous.
- (ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
- (iii) $\mu_{\tilde{A}}(x)$ Strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$
- (iv) $\mu_{\tilde{A}}(x) = \omega$, for all $x \in [b, c]$, where $0 < \omega \leq 1$.

2.4. Definition [6] A fuzzy number $\tilde{A} = (a, b, c, d; \omega)$ is said to be generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \omega \frac{(x-a)}{(b-a)}, & a \leq x < b, \\ \omega & , b \leq x \leq c, \\ \omega \frac{(x-d)}{(c-d)}, & c < x \leq d, \\ 0 & , \text{otherwise.} \end{cases}$$

Arithmetic operations: In this section, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on the universal set of real numbers \mathfrak{R} , are presented [6,7].

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$ are two generalized trapezoidal fuzzy numbers, then the following is obtained.

- (i) $\tilde{A}_1 + \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(\omega_1, \omega_2))$,

- (ii) $\tilde{A}_1 - \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \min(\omega_1, \omega_2))$,

- (iii) $\tilde{A}_1 \otimes \tilde{A}_2 \equiv (a, b, c, d; \min(\omega_1, \omega_2))$, where
 - $a = \min(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2)$,
 - $b = \min(b_1 b_2, b_1 c_2, c_1 b_2, c_1 c_2)$,
 - $c = \max(b_1 b_2, b_1 c_2, c_1 b_2, c_1 c_2)$,
 - $d = \max(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2)$.

- (iv) $\lambda \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; \omega_1), & \lambda > 0, \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; \omega_1), & \lambda < 0. \end{cases}$

Ranking function: An efficient approach for comparing the fuzzy numbers is by the use of ranking function [7,17,19], $\mathfrak{R} : F(\mathfrak{R}) \rightarrow \mathfrak{R}$, where $F(\mathfrak{R})$ is a set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line, where a natural order exists, i.e.,

- (i) $\tilde{A} >_{\mathfrak{R}} \tilde{B}$ if and only if $\mathfrak{R}(\tilde{A}) > \mathfrak{R}(\tilde{B})$
- (ii) $\tilde{A} <_{\mathfrak{R}} \tilde{B}$ if and only if $\mathfrak{R}(\tilde{A}) < \mathfrak{R}(\tilde{B})$
- (iii) $\tilde{A} =_{\mathfrak{R}} \tilde{B}$ if and only if $\mathfrak{R}(\tilde{A}) = \mathfrak{R}(\tilde{B})$

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$ be two generalized trapezoidal fuzzy numbers and $\omega = \min(\omega_1, \omega_2)$. Then

$$\mathfrak{R}(\tilde{A}_1) = \frac{\omega(a_1 + b_1 + c_1 + d_1)}{4} \text{ and}$$

$$\mathfrak{R}(\tilde{A}_2) = \frac{\omega(a_2 + b_2 + c_2 + d_2)}{4} .$$

III. PROPOSED METHOD

The methodology of harmonic mean method is presented as follows.

Step 1: Check whether the given fuzzy transportation problem is balanced or not. If not, balance or by adding dummy row or column with costs are fuzzy zero. Then go to step2.

Step 2: Find the fuzzy harmonic mean for each row and each column. Then find the maximum value among that.

Step 3: Allocate the minimum supply or demand at the place of minimum value of the corresponding row or column.

Step 4: Repeat the step 2 and 3 until all the demands are satisfied and all the supplies are exhausted.

Step 5: Total minimum fuzzy cost = sum of the product of the fuzzy cost and its corresponding allocated values of supply or demand.

IV. NUMERICAL EXAMPLE

To illustrate the proposed method namely, Harmonic Mean Method (HMM) the following Fuzzy Transportation Problem is solved

Example 1: Table 1 gives the availability of the product available at three sources and their demand at three destinations, and the approximate unit transportation cost of the product from each source to each destination is represented by generalized trapezoidal fuzzy number. Determine the fuzzy optimal transportation of the products such that the total transportation cost is minimum.

Table 1

	D ₁	D ₂	D ₃	Supply (a _i)
S ₁	(11,13,14,18;.5)	(20,21,24,27;.7)	(14,15,16,17;.4)	13
S ₂	(6,7,8,11;.2)	(9,11,12,13;.2)	(20,21,24,27;.7)	20
S ₃	(14,15,17,18;.4)	(15,16,18,19;.5)	(10,11,12,13;.6)	5
Demand (b _j)	12	15	11	

Using step1, $\sum_{i=1}^3 a_i = \sum_{j=1}^3 b_j = 38$, so the chosen problem is a balanced FTP. Using **Step2 to Step4** we get:

	D ₁	D ₂	D ₃	Supply (a _i)
S ₁	(11,13,14,18;.5) 7	(20,21,24,27;.7)	(14,15,16,17;.4) 6	*
S ₂	(6,7,8,11;.2) 5	(9,11,12,13;.2) 15	(20,21,24,27;.7)	*
S ₃	(14,15,17,18;.4)	(15,16,18,19;.5)	(10,11,12,13;.6) 5	*
Demand (b _j)	*	*	*	

The minimum fuzzy transportation cost is equivalent to $7(11,13,14,18;.5) + 6(14,15,16,17;.4) + 5(6,7,8,11;.2) + 15(9,11,12,13;.2) + 5(10,11,12,13;.6) = (376,436,474,543;.2)$. Therefore, the ranking function $R(A) = 91.45$

Results with normalization process: If all the values of the parameters used in problem.1 are first normalized and then the Problem is solved by using the HMM, then the fuzzy optimal value is $\tilde{x}_0 = (376,436,474,543;1)$.

Results without normalization process: If all the values of the parameters of the same problem.1 are not normalized

and then the Problem is solved by using the HMM, then the fuzzy optimal value is $\tilde{x}_0 = (376,436,474,543;.2)$.

Remark: Results with normalization process represent the overall level of satisfaction of decision maker about the statement that minimum transportation cost will lie between 436 and 474 units as 100% while without normalization process, the overall level of satisfaction of the decision maker for the same range is 20%. Hence, it is better to use generalized fuzzy numbers instead of normal fuzzy numbers, obtained by using normalization process.

Example 2:

	D ₁	D ₂	D ₃	D ₄	(a _i)
S ₁	(11,12,13,15;.5)	(16,17,19,21;.6)	(28,30,34,35;.7)	(4,5,8,9;.2)	8
S ₂	(49,53,55,60;.8)	(18,20,21,23;.4)	(18,22,25,27;.6)	(25,30,35,42;.7)	10
S ₃	(28,30,34,35;.7)	(2,4,6,8;.2)	(36,42,48,52;.8)	(6,7,9,11;.3)	11
(b _j)	4	7	6	12	

Example 3: [12]

	D ₁	D ₂	D ₃	(a _i)
S ₁	(1,4,9,19;.5)	(1,2,5,9;.4)	(2,5,8,18;.5)	10
S ₂	(8,9,12,26;.5)	(3,5,8,12;.2)	(7,9,13,28;.4)	14
S ₃	(11,12,20,27;.5)	(0,5,10,15;.8)	(4,5,8,11;.6)	15
(b _j)	15	14	10	

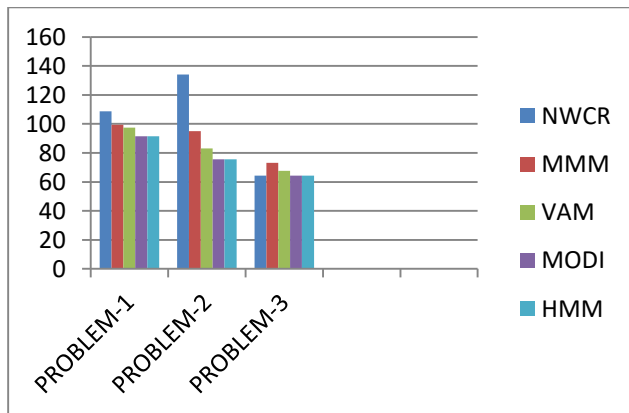
V. COMPARATIVE STUDY AND RESULT ANALYSIS

From the investigations and the results given in Table 2 it clear that HMM is better than NWCR [1], MMM [1] and VAM [21] for solving fuzzy transportation problem and also, the solution of the fuzzy transportation problem is given by HMM is an optimal solution.

Table 2

S.No	ROW	COLUMN	NWCR	MMM	VAM	MODI	HMM
1.	3	3	108.80	99.50	97.50	91.45	91.45
2.	3	4	134.18	95.00	83.00	75.60	75.60
3.	3	3	64.35	73.10	67.60	64.35	64.35

Table 2 represents the solution obtained by NWCR [1], MMM [1], VAM [21], MODI [5] and HMM. This data speaks the better performance of the proposed method. The graphical representation of solution obtained by varies methods of this performance, displayed in graph.



VI. CONCLUSION

From the comparison table 2, we can observe that the optimum solution obtained by the proposed method is less than that of other methods and same that of MODI Method. But, the proposed method is very easy since we have less computation works. So, we can conclude that if we use harmonic mean method to solve transportation problem, we can get global optimum solution in a lesser step.

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