https://dx.doi.org/10.22161/ijcmp.2.2.2 ISSN: 2456-866X

Exact Solutions of Coupled Parallel Resonant Circuits Equations by Decomposition Method

E. U. Agom[1], F. O. Ogunfiditimi[2], I. M. Esuabana[1], B. E. A. Eno[1]

[1]Department of Mathematics, University of Calabar, P. M. B. 1115, Calabar, Nigeria
[2]Department of Mathematics, University of Abuja, Abuja, Nigeria

Abstract— *In this paper, we provide the solution of a modeled parallel resonant circuit using the decomposition method. The parallel resonant circuit also known as resistor (R) in ohms, inductor (L) in Henry and capacitor (C) in farads (RLC) circuit is used in turning radio or audio receivers. The mathematical models are in a system of ordinary differential equations (ODE), which we solve using the Adomian Decomposition Method (ADM). We discovered that this method gave exact solutions as can be obtained using any traditionally known analytic method. These findings are illustrated in three test problems.*

Keywords*— *Adomian Decomposition Method, Resonant Circuits, System of Differential equations.

I. INTRODUCTION

The RLC circuit is one in which the fundamental elements resistor, inductor and capacitors are connected linearly and passively in series or parallel in nature across a voltage supply. A model for parallel RLC circuit using Kirchhoff's voltage and current law is stated in literature with the system of equations. See [7], [8], [9] and [10].

$$\begin{aligned} \alpha' &= \gamma\beta \\ \beta' &= \varpi(\alpha+\beta) \end{aligned} \qquad (1)$$

with $\alpha(0)=0, \beta(0)=\dfrac{E}{R}$

where $\alpha=\alpha(t), \beta=\beta(t), \gamma=\dfrac{R}{L}, \varpi=(-RC)^{-1}$, E is the battery that provide the voltage, R, L and C are as defined previously. Equation (1) has also been used to model a hydraulic system. [8] stated that wires are pipes filled with fluid, the switches correspond to valves and the battery is analogous to a pump which maintains a pressure E. When the pressure is increased it forces the molecules of the fluid to move in a pipe. This paper is divided into sections; in the next section we give the mathematical concept of ADM on a system of differential equations. Subsequently, we give illustration in form of examples using the decomposition method on equation (1) and we conclude.

II. MATHEMATICAL CONCEPT OF ADM ON SYSTEM OF DIFFERENTIAL EQUATIONS

The compact form a system of differential equations can be given as

$$\beta_i' = f_i(t, \beta_1, \beta_2, \beta_3, \dots \beta_n) \qquad (2)$$

where

$i = 1, 2, 3, \dots, n$, β_i' are the derivative for the unknown functions $\beta_1, \beta_2, \beta_3, \dots \beta_n$. Equation 2 is obtained from reduction of nth order ODE.

By ADM as clearly stated by Adomian himself [6] and further simplified by applying in various form in [2], [3], [4], [5], equation (2) is given as

$$L\beta_i' = f_i(t, \beta_1, \beta_2, \beta_3, \dots \beta_n) \qquad (3)$$

where L is a linear operator with an inverse L^{-1}. Applying L^{-1} on both sides of equation (3) we have the following canonical form

$$\beta_i = \beta_i(0) + \int_0^t f_i(t, \beta_1, \beta_2, \dots, \beta_n)dt \qquad (4)$$

By ADM, β_i an the integrand in equation (4) are give as

$$\beta_i = \sum_{j=0}^{\infty} f_{i,j} \qquad (5)$$

$$f_i(t, \beta_1, \beta_2, \beta_3, \dots \beta_n) = \sum_{j=0}^{\infty} A_{i,j}(f_{i,0}, f_{i,1}, f_{i,2}, \dots) \qquad (6)$$

where $A_{i,j}$ are the Adomian polynomials see [5] and [6].

Putting equations (5) and (6) in equation (4), we obtain

$$\begin{aligned} \sum_{j=0}^{\infty} f_{i,j} &= \beta_i(0) + \int_0^t \left[\sum_{j=0}^{\infty} A_{i,j}(f_{i,0}, f_{i,1}, f_{i,2}, \dots)\right]dt \\ &= \beta_i(0) + \sum_{j=0}^{\infty}\left[\int_0^t A_{i,j}((f_{i,0}, f_{i,1}, f_{i,2}, \dots)dt\right] \end{aligned} \qquad (7)$$

From equation (7), the source term and other terms are defined. A comprehensive analysis has been given by [1] and [11] to prove the convergence of equation (5). In the following section, we give illustrations to demonstrate the efficiency of ADM in obtaining exact solutions to system of differential equations. And, to validate the theoretical aspect just discussed in this section.

III. ILLUSTRATIVE EXAMPLE

In this section, we adapt the problems given in [8] and apply the ADM. First we apply the concept of ADM on

https://dx.doi.org/10.22161/ijcmp.2.2.2 *ISSN: 2456-866X*

the governing equation (1). We obtain, on applying the inverse operator, that

$$\alpha' = \gamma L^{-1}(\beta)$$
$$\beta' = \frac{E}{R} + \varpi L^{-1}(\alpha + \beta) \qquad (8)$$

with

$$\alpha_0 = \alpha(0) = 0 \quad \text{and} \quad \alpha_{i+1} = \gamma L^{-1}(\beta_i)$$
$$\beta_0 = \beta(0) = \frac{R}{R} \quad \text{and} \quad \beta_{i+1} = \varpi L^{-1}(\alpha_i + \beta_i)$$

Example 1

In relation to the coupled equation (1), it was stated in [8] that for $L = 4$, $C = 10^{-6}$, $R = 1000$ and $E = 600$ the exact solution are as follows

$$\alpha = 150te^{-500t}$$
$$\beta = (0.6 - 300t)e^{-500t} \qquad (9)$$

We give the Taylors series form of equation (9) as

$$\alpha = 150t - 75E3t^2 + 1875E4t^3 - 3125E6t^4 + \ldots$$
$$\beta = 6E-1 - 6E2t + 225E3t^2 - 5E7t^3 + 78125E5 - \ldots \qquad (10)$$

Applying the concept of ADM in this example we have

$$\alpha_0 = 0, \ \beta_0 = \frac{3}{6}$$
$$\alpha_1 = 150t, \ \beta_1 = -600t$$
$$\alpha_2 = -75000t^2, \ \beta_2 = 225000t^2$$
$$\alpha_3 = 18750000t^3, \ \beta_3 = -50000000t^3$$
$$\alpha_4 = -3125000000t^4, \ \beta_4 = 781250000t^4$$

$\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the first five terms of the first line of equation (10). Similarly, $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ are terms of the second line of equation (10). The result of the analytical solutions and that of ADM are clearly shown in Fig. 1a, 1b, 1c and 1d. The unevenness in the plot is because few terms of the series solution of ADM were applied compare to the whole infinite terms of the exact solution equation (9).

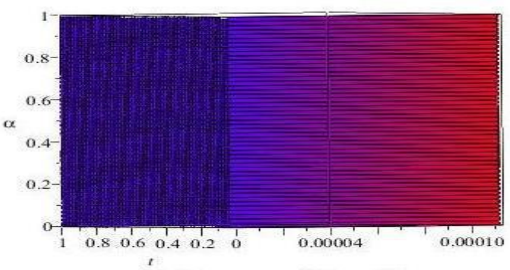

Fig. 1a: α_{Exact} of Example 1

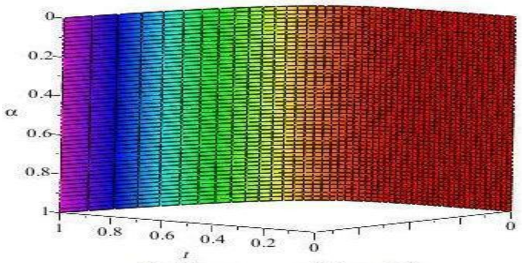

Fig. 1b: α_{ADM} of Example 1

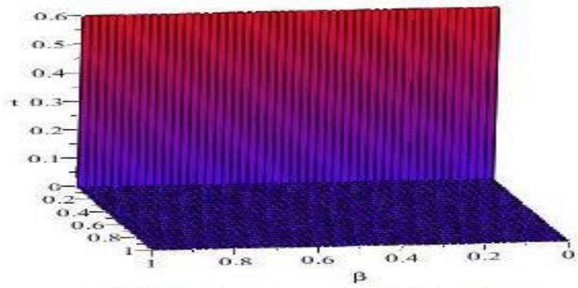

Fig. 1c: β_{Exact} of Example 1

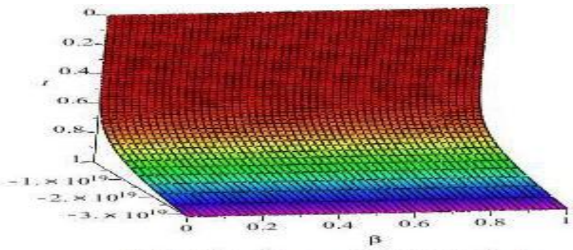

Fig. 1d: β_{ADM} of Example 1

Example 2

Similarly, in relation to the coupled equation (1), it was stated in [8] that for $L = 4$, $C = 10^{-6}$, $R = 600$ and $E = 600$ the exact solution are as follows

$$\alpha = \frac{9(e^{-\frac{500t}{3}} - e^{-1500t})}{80}$$
$$\beta = \frac{-e^{\frac{-500t}{3}} + 9e^{-1500t}}{8} \qquad (11)$$

We give the Taylors series form of equation (11) as

$$\alpha = 150t - 125E3t^2 + \frac{56875E4}{9}t^3 + \wp + \ldots$$
$$\beta = 1 - \frac{5E3}{3}t + \frac{11375E3}{9}t^2 - \frac{5125E7}{81}t^3 + \ldots \qquad (12)$$

where $\wp = -\frac{640625E6}{27}t^4$. Also applying the concept of ADM in this example we obtain the following

$$\alpha_0 = 150t, \ \beta_0 = 1$$
$$\alpha_1 = -125000t^2, \ \beta_1 = -\frac{500}{3}t$$
$$\alpha_2 = \frac{568750000}{9}t^3, \ \beta_2 = \frac{11375000}{9}t^2$$
$$\alpha_3 = -\frac{64062500000}{27}t^4, \ \beta_3 = -\frac{51250000000}{81}t^3$$

$$\alpha_4 = \frac{576640625000000}{81} t^5, \ \ \beta_4 = \frac{57664062500000}{243} t^4$$

$\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ and $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ are the first five terms of α and β in equation (12). Also, Fig. 2a, 2b, 2c and 2d shows the plots of the exact solutions and that of ADM results. The variation in the plots is as a result of using very few terms in the series solution of ADM and infinite terms of the exact solution equation (11).

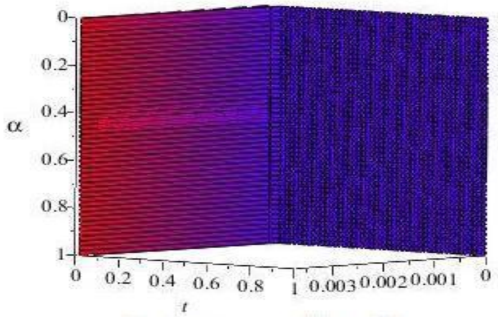

Fig. 2 a: α_{Exact} *of Example 2*

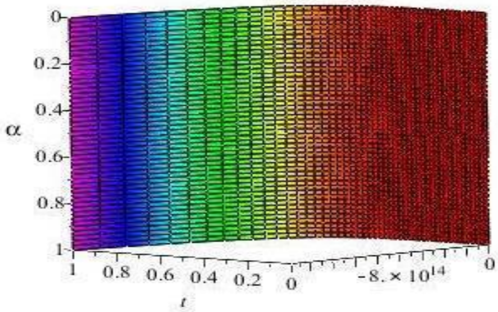

Fig. 2 b: α_{ADM} *of Example 2*

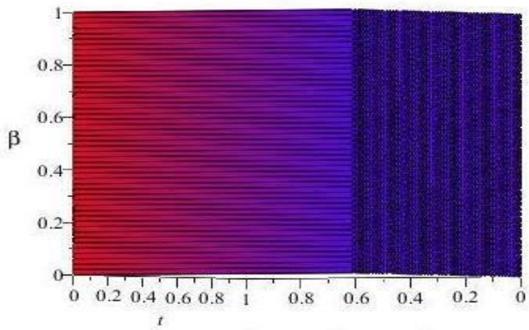

Fig. 2 c: β_{Exact} *of Example 2*

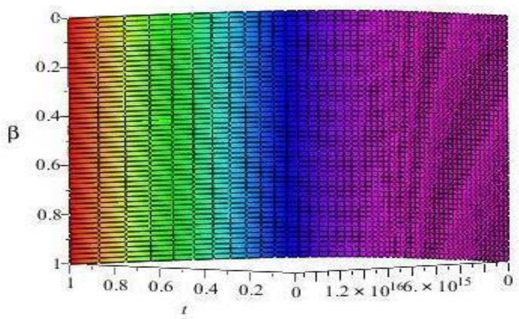

Fig. 2 d: β_{ADM} *of Example 2*

Example 3

Also, in relation to the coupled equation (1), it was stated in [8] that for $L = 2.88$, $C = 10^{-6}$, $R = 1200$ and $E = 600$ the exact solution are as follows

$$\alpha = \frac{1}{2}\sin\left(\frac{1250}{3}t\right)e^{-\frac{1250}{3}t}$$
$$\beta = \frac{1}{2}e^{-\frac{1250}{3}t}\left[\cos\left(\frac{1250}{3}t\right) - \sin\left(\frac{1250}{3}t\right)\right] \quad (13)$$

The series form of equations (13) is given as

$$\alpha = \frac{625}{3}t - \frac{781250}{9}t^2 + \frac{976562500}{81}t^3 - \dots$$
$$\beta = \frac{1}{2} - \frac{1250}{3}t + \frac{781250}{9}t^2 - \frac{610351562500}{243}t^4 + \dots \quad (13)$$

And applying the concept of ADM also to this example, we have

$$\alpha_0 = 0, \ \ \beta_0 = \frac{1}{2}$$

$$\alpha_1 = \frac{625}{3}t, \ \ \beta_1 = -\frac{1250}{3}t$$

$$\alpha_2 = -\frac{781250}{9}t^2, \ \ \beta_2 = \frac{781250}{9}t^2$$

$$\alpha_3 = \frac{976562500}{81}t^3, \ \ \beta_3 = 0$$

$$\alpha_4 = 0, \ \ \beta_4 = -\frac{610351562500}{243}t^4$$

$$\alpha_5 = -\frac{152887890625000}{729}t^5, \beta_5 = 72\frac{305175781250000}{9}t^5$$

Sum of $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are the terms of α in equation (13). Similarly, $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ are also the terms of β in equation (13). Similarly, Fig. 3a, 3b, 3c and 3d depict the plots of the analytical solutions equation (13) and ADM solutions. For ADM only few

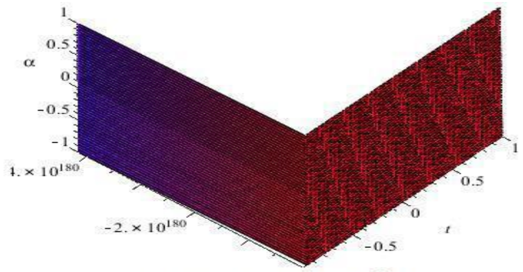

Fig. 3a: α_{Exact} *of Example 3*

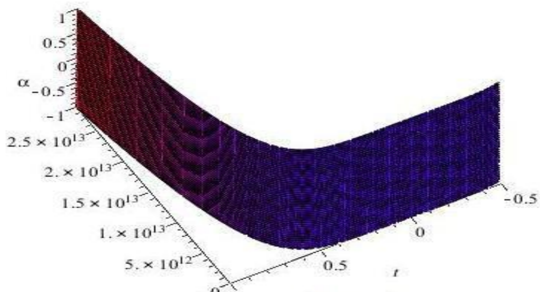

Fig. 3b: α_{ADM} *of Example 3*

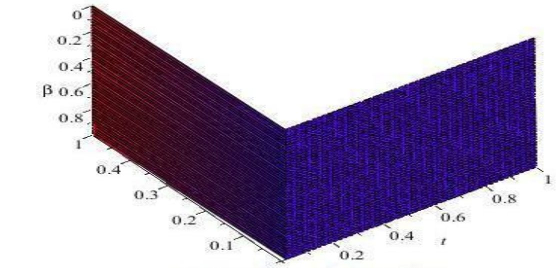

Fig. 3 c: β_{Exact} *of Example 3*

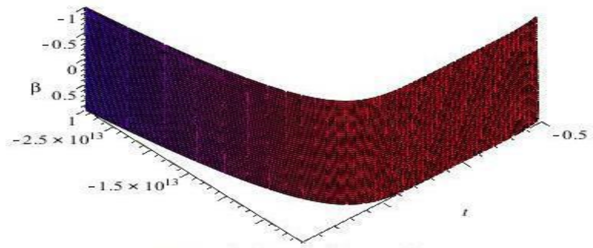

Fig. 3 d: β_{ADM} *of Example 3*

IV. CONCLUSION

We have been able, in this paper, to obtain analytical solution in series form of parallel RLC circuit coupled equation by ADM without any noise term. We showed how ADM concept is applied to coupled first order nth linear equation in n unknown and applied it to a system of two unknown. Our finding has further illustrated the limitless potentials of ADM in providing exact solution to systems of equations. The numerical calculations were made using computer algebra software (Maple) to ensure double precision arithmetic in order to reduce the round-off errors to a minimum.

ACKNOWLEDGEMENTS

The authors are extremely grateful for the supervision of Prof. F. O. Ogunfiditimi, of University of Abuja, despite all odds.

REFERENCES

[1] A. Ahmed, "Convergence of Adomian Decomposition Method for Initial Value Problems", Numerical Method for Partial Differential Equations, vol. 27, pp. 749 – 752. 2011

[2] E. U. Agom, F. O. Ogunfiditimi, E. V. Bassey "Lobatto-Runge-Kutta Collocation and Adomian Decomposition Methods On Stiff Differential Equations" International Journal of Mathematical Research, vol. 6, No. 2, pp. 53-59, December 2017.

[3] E. U. Agom, F. O. Ogunfiditimi, E. V. Bassey, "Numerical Application of Adomian Decomposition Method to Fifth-Order Autonomous Differential Equations" Journal of Mathematics and Computer Science, vol. 7, No. 3, pp. 554-563, May 2017.

[4] E. U. Agom, F. O. Ogunfiditimi, P. N. Assi, "Multistage Adomian Decomposition Methods for Nonlinear 4th Order Multi-point Boundary Value Problems" Global Journal of Mathematics, vol. 10, No. 2, pp. 675-680, July 2017.

[5] E. U. Agom, F. O. Ogunfiditimi, P. N. Assi, "On Adomian Polynomial and its Application to Lane-Emden Type of Equation" International Journal of Mathematical Research, vol. 6, No. 1, pp. 13-21, April 2017.

[6] G. Adomian, Solving Frontier Problems in Physics: The Decomposition Method, New York, Springer

[7] I. Gregory, K. Arkadiy, B. Sam, "An RC Load Model of Parallel and Series Parallel Resonant DC-DC Converter with Output Filter" IEEE Transactions On Power Electronics, vol. 14, No 3, pp. 515–521, May 1999.

[8] L. H. Loomis "Ordinary Differential Equations: Introductory and Intermediate Courses Using Matrix Methods", Adddison-Wesley Publishing Company, Boston, 1971.

[9] T. W. Tushar, S. B. Votthal, "Mathematical Model, Design and Analysis of LLC-T Series Parallel Resonant Converter" IOSR Journal of Electronic and Communication Engineering (IOSR-JECE), vol. 14, pp. 19–25,

[10] W. Janusz, J. Agnieszka, "Analysis of Parallel Resonance Circuit with Supercapacitor" Pozan University of Technology Academic Journal, No 77, pp. 93–99, 2014.

[11] Y. Cherruault, G. Adomian, K. Abbaoui, R. Rach, "Further Remarks on Convergence of Decomposition", International Journal of Bio-Medical Computing, vol. 38, pp. 89-93, 1995.