Open Access

*International journal of Chemistry, Mathematics and Physics (IJCMP)*
*[Vol-3, Issue-6, Nov-Dec, 2019]*
*https://dx.doi.org/10.22161/ijcmp.3.6.1*
*ISSN: 2456-866X*

# Modeling failure waves in brittle materials

Yehuda Partom

Retired from RAFAEL, P.O. Box 2250, Haifa, Israel
ypartom@netvision.net.il

***Abstract***— *Failure waves in glass were first observed in tests some 30 years ago, with a wave velocity of 1.5-2.5 km/s. In spite of the long time since then, some essential questions concerning failure waves remained unanswered. These are: 1) what is the formation mechanism of failure waves; 2) what is the propagation mechanism of failure waves; and 3) what are the kinetics of the failure process? In the past failure wave researchers assumed that material damage starts from the boundary. But in a recent experimental work on glass [9] the investigators observed that the glass starts to fail within the material behind the shock front, and not from the boundary. This seemingly small change in the way failure waves are started makes it possible to predict the mechanics of failure wave formation and propagation, using existing failure models for brittle materials. We're using here a dynamic failure model for brittle materials that we've developed in recent years [10]. To get a failure wave that lags behind the shock front, we assume in that the rate of damage accumulation behind the shock front decreases exponentially with distance from the boundary. This is a plausible assumption because opening pores and cracks would become more difficult with distance from the boundary. And indeed, using this assumption we get a failure wave that propagates slower than the shock and at an approximately constant velocity.*

***Keywords*— brittle materials, waves in glass, shear stress.**

## I. INTRODUCTION

Failure waves in glass were first observed in planar impact tests in 1991 [1]. Since then the existence of failure waves has been verified at different laboratories and for various glasses [2-7]. From the test results we can draw the following picture: 1) for a shock of amplitude above about 0.5 HEL that goes into the glass, a failure wave spreads in the glass behind the initial shock; 2) the velocity of the failure wave is 1.5-2.5km/s; 3) behind the failure wave the spall strength goes down to zero and the shear stress decreases. The decrease of the shear stress comes about through the decrease of the lateral stress, while the longitudinal stress stays almost the same.

But important issues concerning failure waves still remained unanswered. Some of those are: 1) what is the mechanism responsible for the formation of failure waves; 2) what is the mechanism controlling the propagation of failure waves; and 3) what are the kinetics of the failure process. As long as there are no answers to those questions it's not possible to construct a model to predict the formation of failure waves with a simulation code.

In [8] we constructed a model that makes it possible to predict a failure wave in planar impact. But because we didn't have the answers to the above questions, we postulated the existence of the failure wave and its velocity. Our model predicted correctly the histories of the stress components behind the shock and within the failure wave.

In the articles [2-7] mentioned above they always assume that formation of material damage (pores and cracks) starts at the longitudinal boundary. And this might be the main reason why so many researchers were not able for so many years to propose a model for predicting the formation and the velocity of propagation of failure waves.

In [9] they conducted tests for which they could observe the formation and propagation of failure waves in glass. The test diagnostics that they used include a PDV laser interferometer, which made it possible to observe closely the dynamics of the damage development in the glass. From the results they concluded that the glass starts to fail inside the glass behind the shock wave, and not from the boundary. This may seem like a small change in the process of damage accumulation, but it has a large effect in terms of modeling the formation and propagation of the failure wave.

In [10] we present our brittle materials failure model that we've developed for some years. Unlike other models for the dynamic response of brittle materials [11], our model is based on the overstress approach, which considers the rate of loading relative to the rate of damage accumulation.

In what follows we use our brittle materials failure model, together with the assumption that damage starts within the

International journal of Chemistry, Mathematics and Physics (IJCMP)
[Vol-3, Issue-6, Nov-Dec, 2019]
https://dx.doi.org/10.22161/ijcmp.3.6.1
ISSN: 2456-866X

bulk of the material, to model the formation and propagation of failure eaves.

## II. FAILURE MODEL FOR BRITTLE MATERIALS

For brittle materials we define (on the macro scale) a fracture threshold curve, or a damage accumulation threshold curve, in the P,S plane (P=pressure and S=equivalent stress, which is a measure of the shear stress). Such a threshold curve is similar in nature to a plastic flow stress curve for ductile materials.

When the state point of a computational cell (or a representative volume element) containing a brittle material is on the fracture threshold curve, the material there does not fail right away, but starts to accumulate damage and loose its strength gradually. The state point can therefore protrude out of the fracture threshold curve. We call such a description of the material response an overstress approach. The overstress approach to dynamic response recognizes that various threshold crossing processes are not instantaneous, but require some finite time. For a dynamic response to fast loading (like shock loading), a threshold crossing may therefore lag behind the loading process.

When the state point of some computational cell is beyond the fracture threshold curve, pores and cracks are being formed in the material there. In a macroscopic response model we represent those pores and cracks in terms of a quantity known as the amount of damage, or just damage, which we denote by D. It's customary to define D as varying between 0 and 1.

When D=0, the material is intact.

When 0<D<1, the material is porous and fractured, and its response is still elastic, but with reduced stiffness.

When D=1, the material is fractured to such an extent that it flows plastically under shear stress loading, and it is referred to as failed.

We assume here, and this is the usual assumption [11], that the response of failed brittle materials is similar to that of granular materials. Granular materials oppose plastic flow as a result of friction between grains that are in contact and move past each other. And because of Coulombs law for resistance to friction, it's customary to assume that the shear stress response of a failed brittle material increases linearly with pressure.

When the stress point of a computational cell is beyond the fracture threshold curve (overstress state), the amount of damage increases with time. We also assume that the rate of damage accumulation increases with the amount of overstress.

In addition, it is customary to assume [11] that when 0<D<1 and the amount of damage increases, the fracture threshold curve approaches the totally failed curve, and the amount of approach is given by:

$$S_D\left(P\right)=\left(1-D\right)S_i\left(P\right)+DS_f\left(P\right) \qquad (1)$$

where the indexi stands for intact, and the index f stands for failed.

In Fig. 1 we show the various curves mentioned above in the S,P plane.

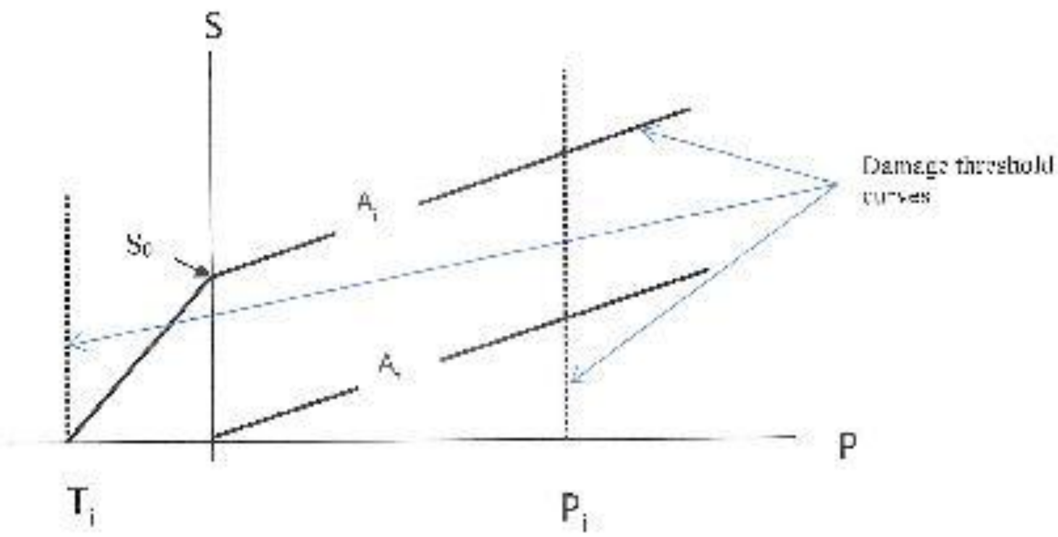


*Fig.1: Damage threshold curves for brittle materials. $A_i$ and $A_f$ are the slopes of the initial fracture threshold curve and the fully failed curve (here straight lines). The lines $P_i$ and $T_i$ (parallel to the S axis) are fracture threshold curves for pure pressure and tension, which we don't deal with here.*

International journal of Chemistry, Mathematics and Physics (IJCMP)
[Vol-3, Issue-6, Nov-Dec, 2019]
https://dx.doi.org/10.22161/ijcmp.3.6.1
ISSN: 2456-866X

From lack of any concrete information we assume here that the rate of damage accumulation increases linearly with the amount of overstress relative to the current fracture threshold curve:

$$\dot{D} = A_D\left(S - S_D\right) \quad S > S_D \quad D < 1 \qquad (2)$$

Later we show that when using this assumption by itself, we don't get a constant velocity failure wave. We get instead a failure process that moves with the shock velocity and lags behind the shock by a constant amount.

## III. PLANAR IMPACT SIMULATIONS

Next we use the model described above in planar impact simulations.

The material in our simulations is glass with a Gruneisen EOS and the following parameters:

$\rho_0$=2.23g/cc, $C_H$=3.88mm/μs, $S_H$=1.63, Γ=1, ρΓ=const., G=24.GPa, where $\rho_0$=initial density, $C_H$,$S_H$=Hugoniot curve parameters in the velocities plane, Γ=Gruneisen parameter and G=shear modulus.

In addition we're using the following failure model parameters:

$S_0$=3.5GPa, $A_i$=$A_f$=0.2/GPa, $P_i$=29GPa, $T_i$=-1.GPa, $A_D$=0.5/GPa/μs.

The ingoing shock is 10GPa. The glass thickness is 10mm, and the resolution is 10 cells/mm.

In Fig. 2 we show damage histories at several locations into the sample.

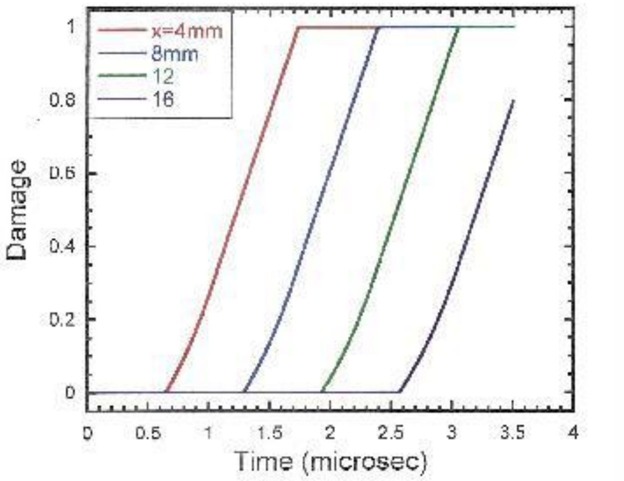


*Fig.2: Damage histories at different locations into the sample computed with the damage and failure model described above.*

From Fig. 2 we see that the damage histories at different locations into the sample are identical. If we therefore choose a certain damage level as representing the failed material, we get that onset of material failure lags by the same distance behind the shock for each location into the sample. In other words we get that the failure wave velocity is the same as the shock velocity. From this result we deduce that there has to be an additional factor controlling the rate of damage accumulation.

The additional factor that we propose is the distance from the boundary. We propose that the rate of damage accumulation decreases as the distance from the boundary increases, which we explain as follows:

As mentioned above, damage means the formation of pores and cracks on the mesoscale, and formation of pores and cracks at a certain location requires material motion in all directions from that location. The more a location is distant from the sample boundaries, it's harder for the material there to move and enable the formation of pores and cracks. Accordingly we define the coefficient $A_D$ in Eq. (2) to be decreasing with distance from the boundary, and our corrected equation for damage accumulation becomes:

$$\dot{D} = A_D \exp\left(-x/x_{ref}\right)\left(S - S_D\right) \qquad (3)$$

where x is the distance from the boundary,$x_{ref}$ is a parameter controlling the dependence of the rate of damage accumulation on distance from the boundary, and where our use of the exponential function is based on our intuition. Fig. 3 below is similar to Fig. 2, but with $x_{ref}$=5.mm.

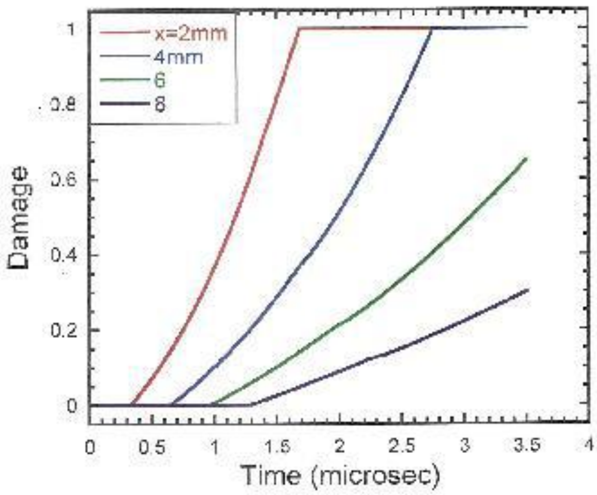


*Fig.3: Histories of the amount of damage at different locations into the sample, with the corrected damage accumulation rate (Eq. (3)).*

We see from Fig.3 that as the distance from the boundary increases, the rate of damage accumulation decreases. The result is (as shown later) that the propagation velocity of a constant damage level is lower than the shock velocity.

https://dx.doi.org/10.22161/ijcmp.3.6.1
ISSN: 2456-866X

In Fig. 4 we show the propagation velocity of the 0.5 damage level, and for two values of $x_{ref}$.We see from Fig. 4 that indeed, the corrected equation for the rate of damage accumulation produces a failure wave velocity that is lower than the shock velocity.

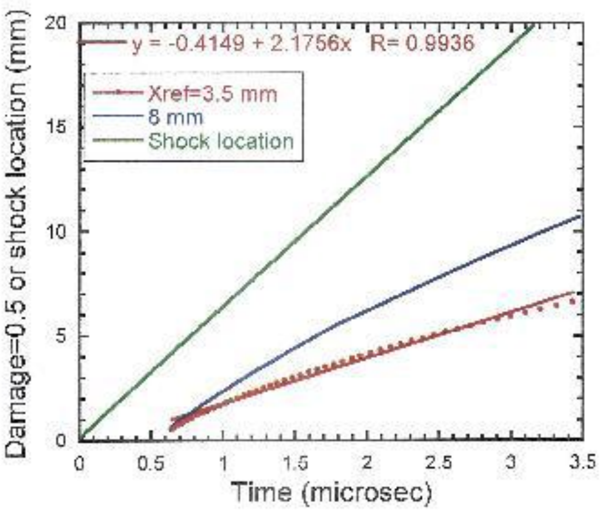


*Fig.4: x(t) curves. Green: shock front. Blue and red: location for which damage level is 0.5, blue for $x_{ref}$=8mm and red for $x_{ref}$=3.5mm. The average slope of the red curve is 2.2km/s, about the same as in the tests.*

## IV. SUMMARY

Failure waves in glass were first observed in tests some 30 years ago. The main characteristics of those failure waves are: 1) the failure wave velocity is 1.5-2.5km/s; 2) behind the failure front the spall strength goes down to zero, and the shear strength decreases somewhat. The shear strength decrease is caused mainly through the decrease of the lateral stress component, while the longitudinal component stays almost unchanged.

In spite of the long time since the discovery of failure waves in glass, some essential questions concerning failure waves remain unanswered. These are: 1) what is the formation mechanism of failure waves; 2) what is the propagation mechanism of failure waves; and 3) what are the kinetics of the failure process? As long as there are no answers to those questions, it's not possible to construct a model that can predict failure waves, using a hydrocode simulation.

In the past we constructed a model to predict failure waves in planar impact. But because we didn't have answers to the above questions, we assumed apriori the existence of a failure wave and its velocity. Our model predicted correctly the histories of stress components behind the shock wave and in the failure wave.

In the past all failure wave researchers assumed that material damage starts from the boundary, and may be this was the main reason that for so many years no one was able to propose a model to predict correctly the formation and propagation of failure waves. But in a recent experimental work on glass [10] they observed that the glass starts to fail within the material behind the shock front, and not from the boundary. This seemingly small change in the way failure waves are started makes it possible to predict the mechanics of failure wave formation and propagation, using existing failure models for brittle materials.

We're using here a dynamic failure model for brittle materials that we developed in recent years [10]. We use the overstress approach tofast dynamic loading, and describe the dynamic process of damage accumulation in a computational cell from a no damage state (intact) to a fully damaged state (failed). We also assume that the response of a failed material is similar to that of a granular material (strength increases with pressure).

In our original model we assume that the rate of damage accumulation increases with the overstress relative to the fracture threshold curve, irrespective of location. Applying this to failure wave tests, where a planar impact enters from the boundary, we don't get a failure wave. We get instead the same damage accumulation process everywhere behind the shock front.

Therefore, to get a failure wave that lags behind the shock front, we assume in addition that the rate of damage accumulation decreases exponentially with distance from the boundary. This is a plausible assumption because opening pores and cracks becomes more difficult with distance from the boundary. And indeed, using this assumption we get failure waves that propagates slower than the shock and at an approximately constant velocity (see Fig. 4).

## REFERENCES

[1] S.V. Rasorenov et al., The fracture of glass under high pressure impulsive loading, High Press. Res. 6, 225 (1991).
[2] G.I. Kanel et al., The failure waves and spallations in homogeneous brittle materials, Shock Compression in Condensed Matter, 451 (1991).
[3] S.J. Bless et al., Failure waves in glass, J. Amer. Cer. Soc. 75, 1002 (1992).
[4] Raiser et al., Plate impact response of ceramics and glasses, J. Appl. Phys. 75, 3862 (1994).
[5] N.K. Bourn and Z. Rozenberg, The dynamic response of soda-lime glass, Shock Compression in Condensed Matter 567 (1995).
[6] N.S. Brar and S.J. Bless, Failure waves in glass under dynamic compression, High Press. Res. 10, 73 (1992).

*International journal of Chemistry, Mathematics and Physics (IJCMP)*
*[Vol-3, Issue-6, Nov-Dec, 2019]*
*https://dx.doi.org/10.22161/ijcmp.3.6.1*
*ISSN: 2456-866X*

[7] N.K. Bourne et al., Impact and penetration of borosilicate glass, Shock Waves Physics Group, Cavendish Lab, Cambridge UK (1997).
[8] Y. Partom, Modeling failure waves in glass, Int. J. Impact Eng. 21, 791 (1997).
[9] S. Chocron et al., Experimental and computed results investigating time dependent failure in a borosilicate glass, Shock Compression of Condensed Matter, AIP conf. Proc. 100005-1-4 (2015).
[10] Y. Partom. Modeling high rate stress upturn for brittle materials, Shock Compression of Condensed Matter (2019).
[11] G.R. Johnson and T.J. Holmquist, A computational model for brittle materials to large strains, high strain rates, and high pressures, in Shock Waves and high strain rate phenomena in materials (1992).